Statistical Process Control

Basic SPC

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Omnex provides training, consulting and software solutions to the international market with offices in the USA, Canada, Mexico, China (PRC), Germany, India, the Middle East, and SE Asia. Omnex offers over 400 standard and customized training courses in business, quality, environmental, food safety, laboratory and health & safety management systems worldwide.

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Course Objectives

- Identify the different uses of basic variables control charts.
- Explain common and special causes.
- Relate within and between variation to common and special causes.
- Explain the relationship between process control and process capability.
- Explain the relationship between C and P indices, and the different methods of estimating standard deviations.
- Identify appropriate uses for C_p, C_{pk} and P_p, P_{pk}
- Explain the relationship between the capability indices to determine process improvement actions.

Agenda

- Course Overview and Introductions
- Chapter 1 – SPC Background
  - **Breakout Exercise 1: Analyzing Data**
- Chapter 2 – Introduction to Control Charts
- Chapter 3 – Basic Variable Control Charts
  - **Breakout Exercise 2: Plotting Data**
  - **Breakout Exercise 3: Control Charts**
  - **Breakout Exercise 4: $\overline{X}$ & MR Charts**
- Chapter 4 – Basic Attribute Control Charts
- Chapter 5 – Analyzing Control Charts
  - **Breakout Exercise 5: Interpreting Control Charts**
- Chapter 6 – Capability Analysis
  - **Breakout Exercise 6: Calculating Indices**

A BRIEF INTRODUCTION TO OMNEX

Omnex Introduction

- International consulting, training and software development organization founded in 1985.
- Specialties:
  - Integrated management system solutions.
  - Elevating the performance of client organizations.
  - Consulting and training services in:
    - Quality Management Systems, e.g. ISO 9001, IATF 16949, AS9100, QOS
    - Environmental Management Systems, e.g. ISO 14001
    - Health and Safety Management Systems, e.g. OHSAS 18001
- Leader in Lean, Six Sigma and other breakthrough systems and performance enhancement.
  - Provider of Lean Six Sigma services to Automotive Industry via AIAG alliance.

About Omnex

- Headquartered in Ann Arbor, Michigan with offices in major global markets.
- In 1995-97 provided global roll out supplier training and development for Ford Motor Company.
- Trained more than 100,000 individuals in over 30 countries.
- Workforce of over 400 professionals, speaking over a dozen languages.
- Former Delegation Leader of the International Automotive Task Force (IATF) responsible for ISO/TS 16949.
- Served on committees that wrote QOS, ISO 9001:2000, QS-9000, ISO/TS 16949 and its Semiconductor Supplement, and ISO IWA 1 (ISO 9000 for healthcare).
- Member of AIAG manual writing committees for FMEA, SPC, MSA, Sub-tier Supplier Development, Error Proofing, and Effective Problem Solving (EPS).

Omnex is headquartered and operates from the United States through offices in Michigan.

The company maintains international operations in many countries to provide comprehensive services to clients throughout Western Europe, Latin America and the Pacific Rim.

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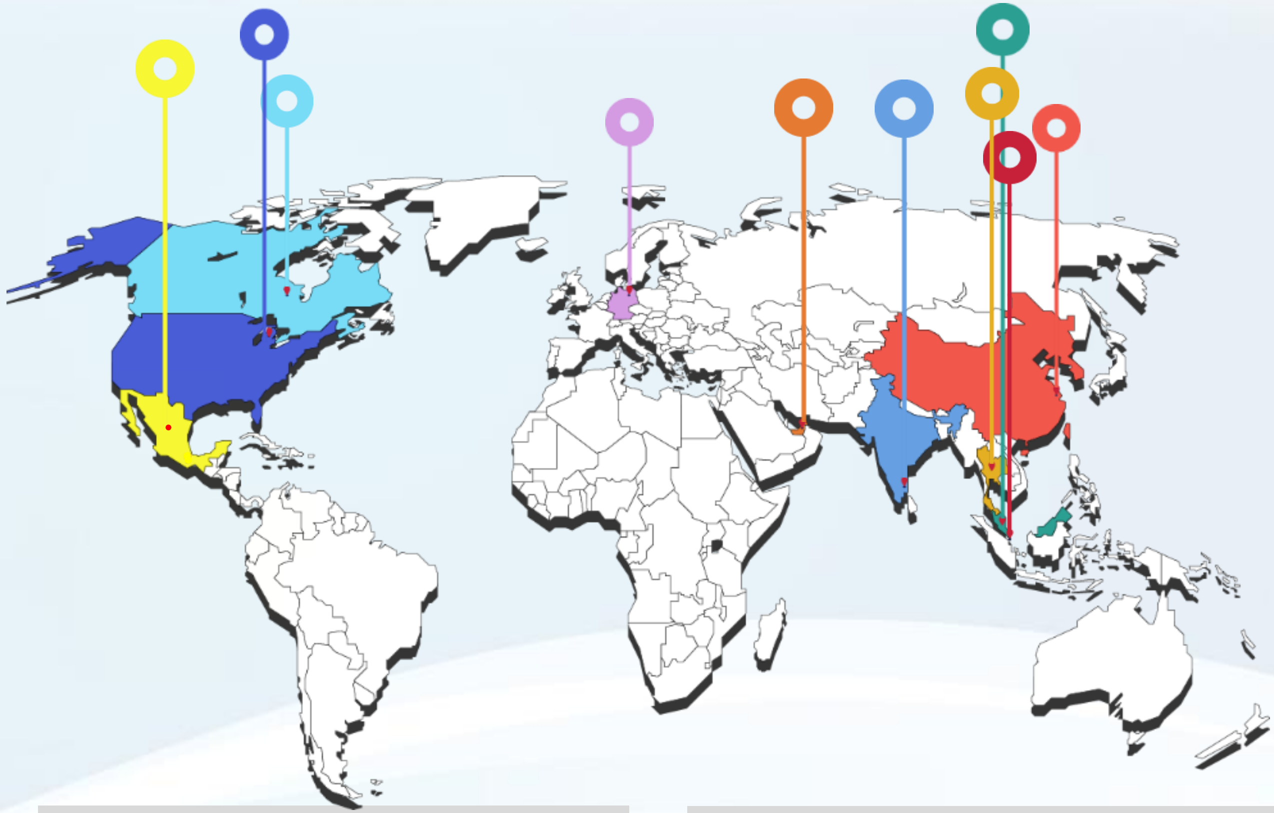

- **Omnex Global Head Quarters** (Michigan, USA)
  West Coast Operations (San Jose, CA)
- **Asia Pacific HQ** (Chennai, Pune, Delhi, Bangalore)
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- **Thailand (Bangkok)**
- **Mexico (Monterrey)**
- **Singapore**
- **Malaysia (Kuala Lumpur)**

Rules of the Classroom

- ✓ Start and end on time
- ✓ Return from breaks and lunch on time
- ✓ All questions welcome
- ✓ Your input is valuable and is encouraged
- ✓ Don’t interrupt others
- ✓ One meeting at a time
- ✓ Listen – and respect others’ ideas
- ✓ No “buts” – keep an open mind
- ✓ Cell phones & pagers off or silent mode
- ✓ No e-mails, texting or tweeting during class
- ✓ If you must take a phone call or answer a text please leave the room for as short a period as possible

Icebreaker

- Instructor Information:
  - Name
  - Background

- Student Introductions: OMNEX
  - Name
  - Position / Responsibilities
  - What is your involvement in the new product development process?
  - What are your experiences with SPC?
  - Please share something unique and/or interesting about yourself.

Chapter 1

SPC Background

Chapter 1: SPC Background – What We Will Cover

Learning Objectives

At the end of this chapter, you will be able to describe:

- The voice of the customer and the voice of the process
- The purpose of SPC
- The difference between prevention and detection
- Process control and process capability
- Over-control
- Location statistics and spread statistics

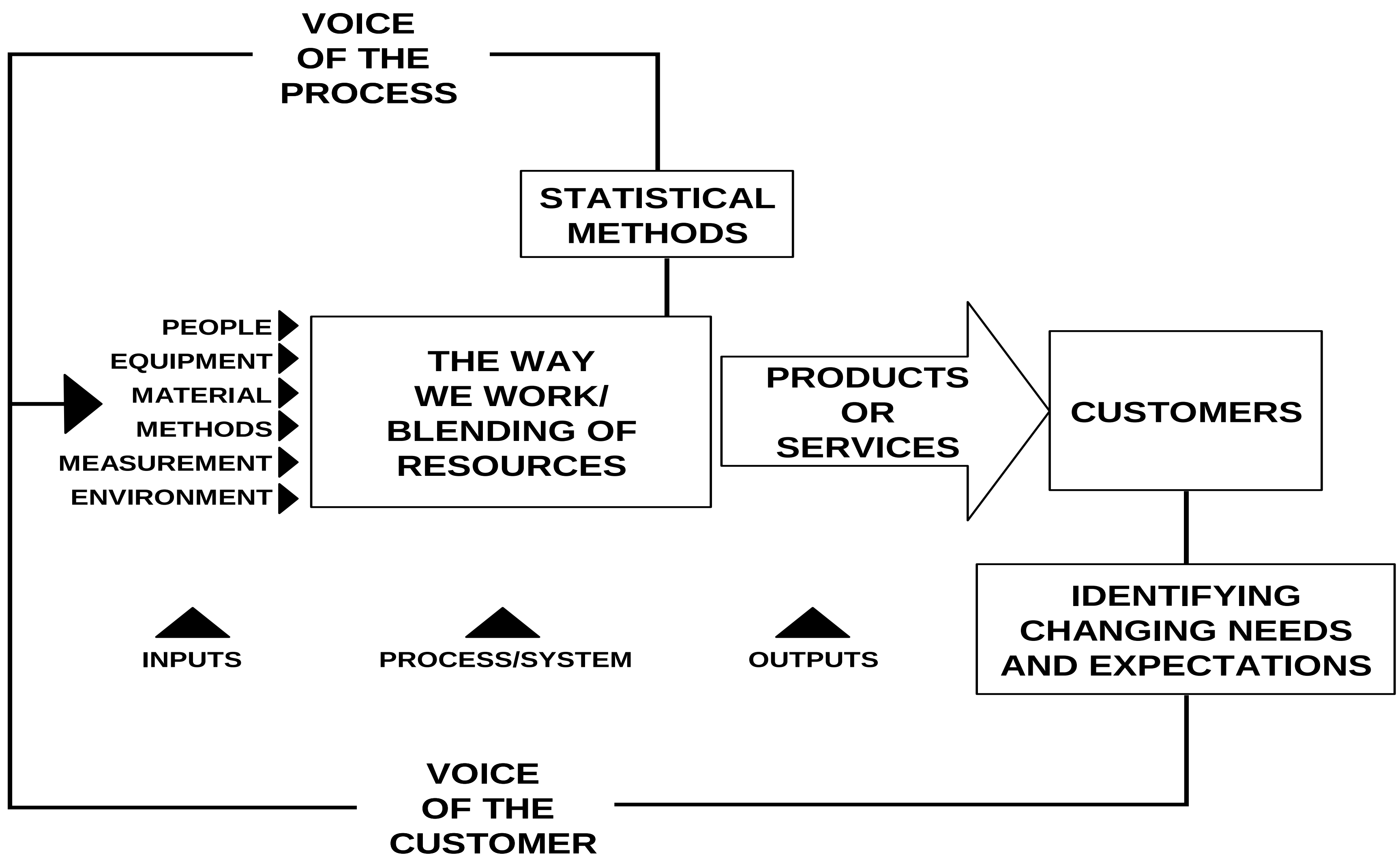
Chapter Agenda

- Process Fundamentals
- Prevention vs. Detection
- Process Variation
- Control vs. Capability
- Basic Statistics
- **Breakout Exercise 1**
- Normal Distribution
- Central Limit Theorem

"Control charts are easy to construct, so they are widely used. But there are surprisingly few really useful charts."

Ishikawa (1982), *Guide to Quality Control*

Fundamentals of Process Control



PREVENTION VS. DETECTION

Prevention Versus Detection

Detection – Tolerates Waste

Prevention – Avoids Waste

Strategy of Prevention

- A prevention strategy sounds sensible – even obvious – to most people.
- It is easily captured in such slogans as, "*Do it right the first time*."

However, slogans are not enough

- What is required is an understanding of the elements of a statistical process control system.

"By What Means"
W.E. Deming

Strategy of Prevention

We need to understand:

- What is meant by a process control system?
- How does variation affect process output?
- How can statistical techniques tell whether a problem is local in nature or involves broader systems?
- What is meant by a process being in statistical control, and what is meant by a process being capable?
- What is a continual improvement cycle, and what part can process control play in it?
- What benefits can be expected from using control charts?

PROCESS VARIATION

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Total Process Variation

Total Process Variation

Common Cause Variation

Within Variation

- Inherent Variation
- Natural Variation
  - the process capability
- ***All*** recipients see this variation
- Occurs within the subgroups

Special Cause Variation

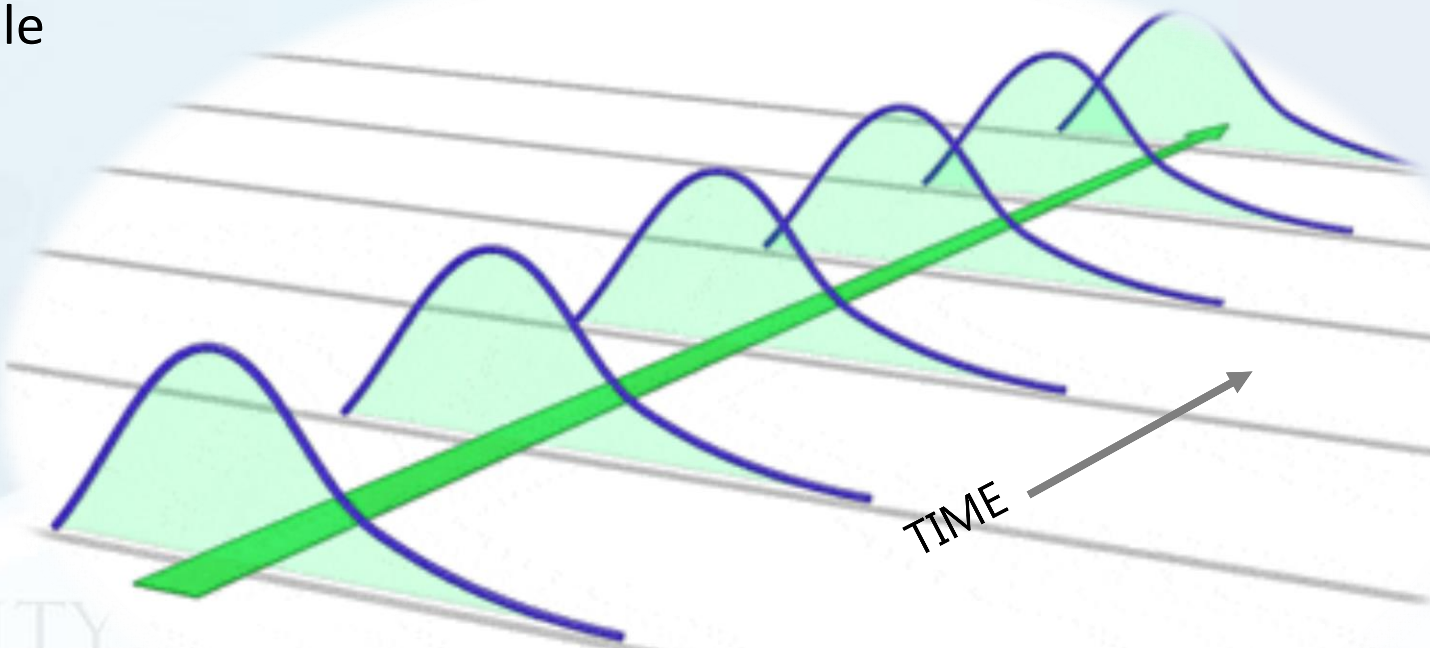
Between Variation

- "Un"Natural Variation
- *Only some* recipients see this variation
- Something **CHANGES** in the process
- Occurs between the subgroups

Understanding Process Behavior

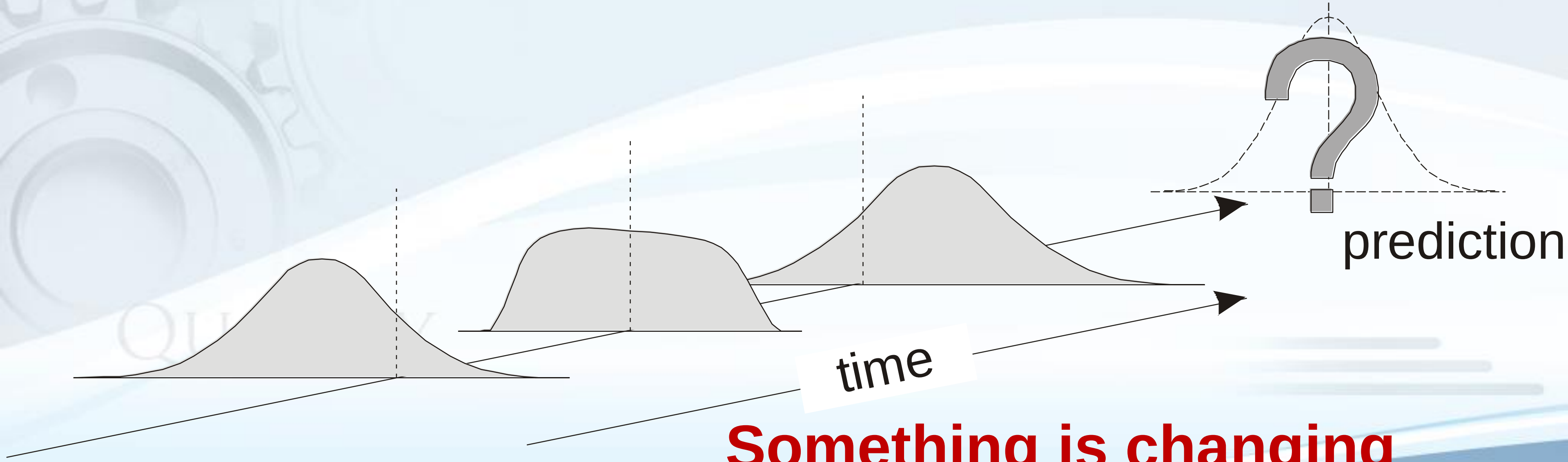
If only common causes of variation are present, the output of a process forms a distribution that is:

- stable over time and
- predictable



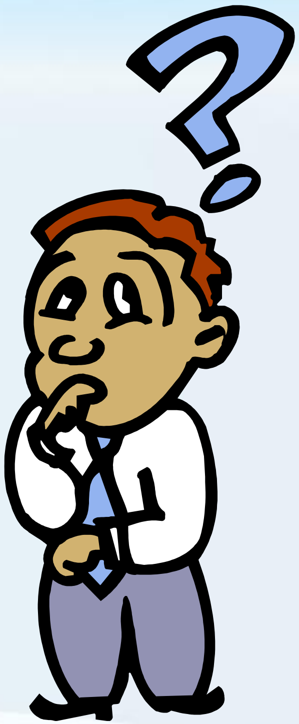
Understanding Process Behavior

If special causes of variation are present, the process output is not stable over time; we are ***unable to predict the results***



Something is changing

Why is This Important?



- Predictions require
  - "*stable*" processes
  - only common cause variation
  - processes in statistical control

The actions we take are different depending upon if the variation is due only to common causes or include one or more special causes.

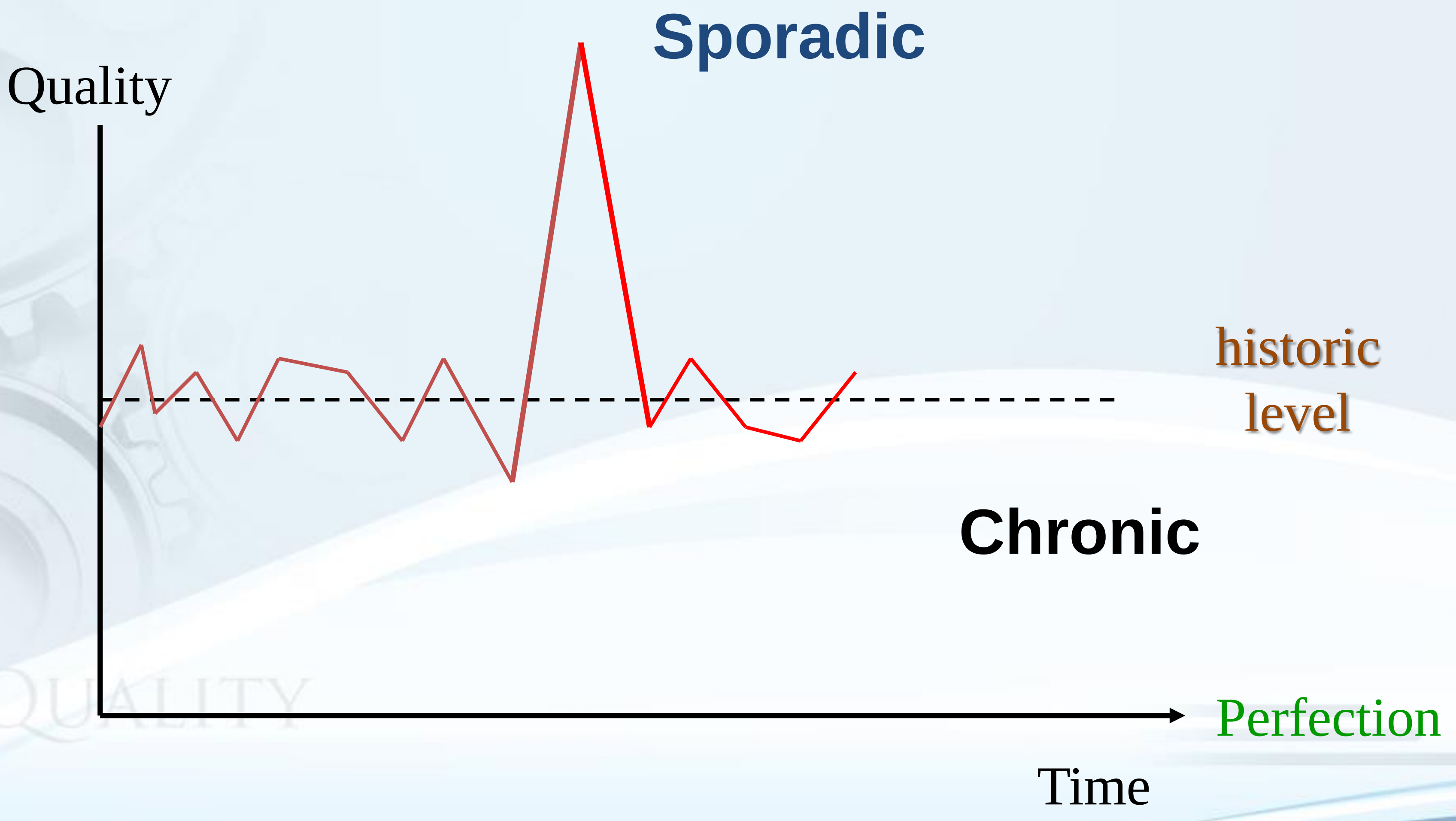
Two Mistakes

Mistake #1: To react to an outcome as if it came from a special cause when actually it came from common causes of variation.

"Over control" = Tampering

Mistake #2: To react to an outcome as if it came from common causes of variation when actually it came from a special cause.

VARIATION: Sporadic or Chronic



CONTROL VS. CAPABILITY

Control vs. Capability

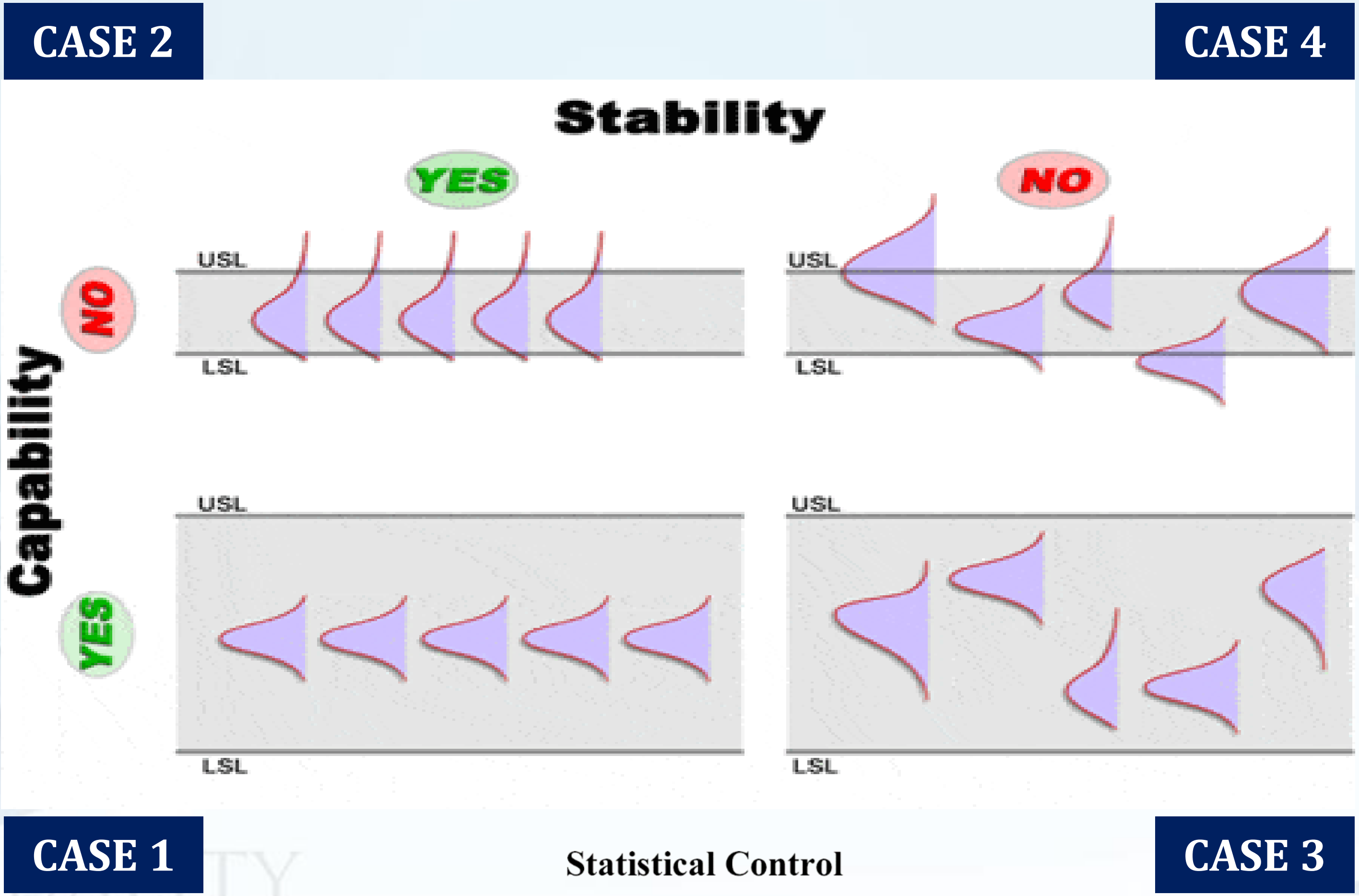
- Control deals with the sources of variation affecting the process.
- Capability is the variation left when all special causes of variation are removed.
  - The variation due only to common causes.

Capability vs. Performance

- **Process capability** is determined by the variation that comes from common causes.
  - This generally represents the best performance of the process itself and is demonstrated when the process is being operated in a state of statistical control.

- Customers, internal or external, are typically more concerned with the **process performance** – that is, the overall output of the process, special and common causes, and how it relates to their requirements (defined by specifications), irrespective of the process variation.

Control vs. Capability



Statistical Control

		In-Control	Out-of-Control
Capability	Acceptable	*Case 1*	*Case 3*
	Unacceptable	*Case 2*	*Case 4*

BASIC STATISTICS

QUALITY

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Variable Data

In order to make use of any statistical tools, we must be able to make measurements – this is called **Variable Data**.

- **Variable Data** is the input for drawing a picture of the variation in process (or even in a product or service).
- Data is grouped into sets to see if there are any patterns, such as the shape of the distribution (i.e., symmetry, skewness, flatness), the center of the distribution (i.e., mean, mode, median), and the spread of data (i.e., range, standard deviation.)

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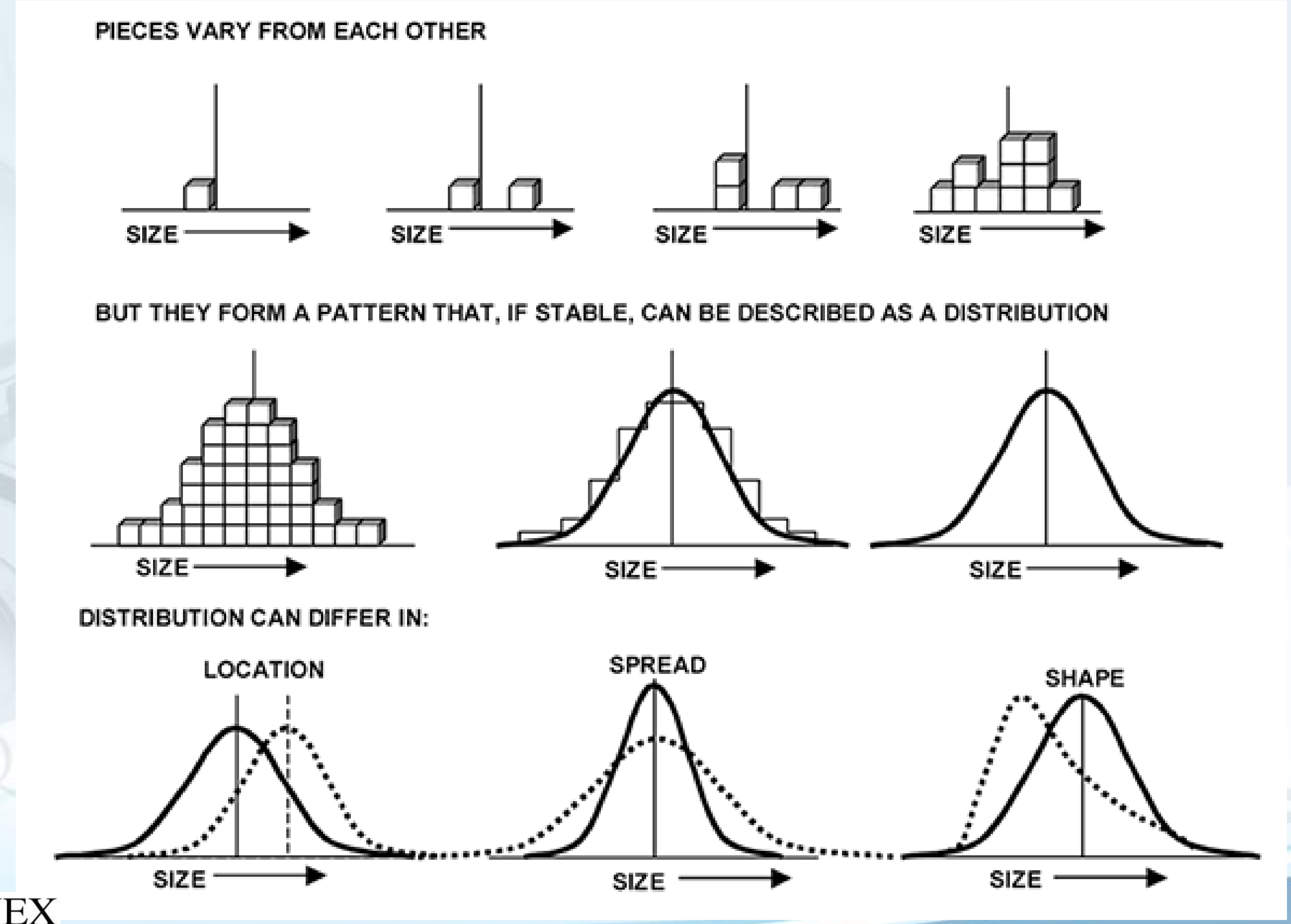
Understanding Process Behavior



Summarizing Information Contained in Sets of Numbers

- Statisticians have developed numerous tools to answer the questions…
  - Where is the “*center*” of a distribution of numbers?
  - To what extent are the numbers concentrated around, or spread out from, the center?
  - What is the “*shape*” (or distribution) of the numbers?

These Tools Are…

- Measures of *central tendency* or *location* of the observations:
  - Mean (Average)
  - Median
  - Mode
- Measures of *variation* or spread of the observations:
  - Range
  - Variance
  - Standard Deviation
- Information about the *shape* of the distribution of the observations:
  - Symmetry (Similarity to a normal distribution)
  - Skewness
  - Kurtosis (Flatness or Heavy Tails)

Average

The average is the most commonly used measure of location in SPC.

$$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$where$$

$$n = number\ of\ samples$$

$$x_i = value\ of\ the\ ith\ sample$$

Range

The range is the most commonly used measure of variability in SPC.

The range takes into account only the high and low values in a set of values.

$$Range = R = \max\{x_i\} - \min\{x_i\}$$

Standard Deviation

The variance is the most commonly used measure of variability in statistical analysis.

Variance takes into account all the values in a set of items.

$$s^2 = \frac{\sum_{i=1}^{n} \left(x_i - \bar{X} \right)^2}{n-1}$$

$$\bar{X} = \text{the mean of the } x_i \text{ values}$$

$$s = \sqrt{s^2} = \text{the standard deviation}$$

Shewhart's Two Rules for the Presentation of Data:

1. Data should always be presented in such a way that preserves the evidence in the data for all the predictions that might be made from these data.

2. Whenever an average, range, or histogram is used to summarize data, the summary should not mislead the user into taking any action that the user would not take if the data were presented in a time series.

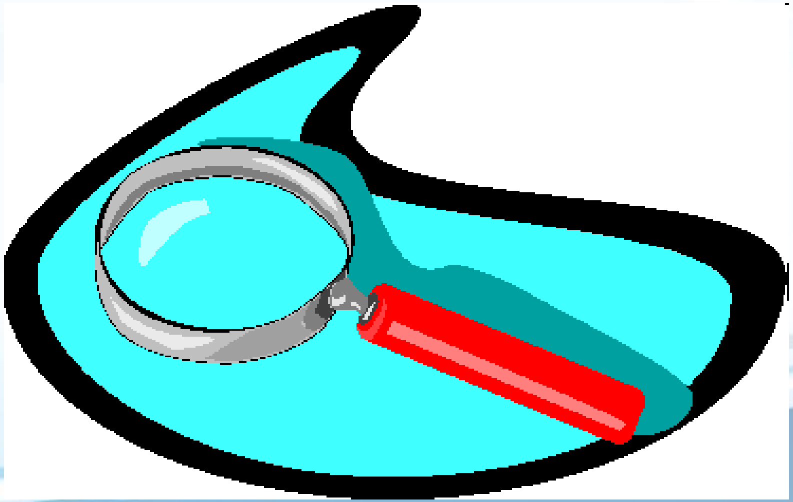
Breakout Exercise 1

Analyzing Data

Breakout Exercise 1: Analyzing Data

Instructions

- Turn to Exercise 1 in your workbook.
- Analyze the collected data.
- Discuss your observations with the team. O-M-N-E-X
- Share your findings with the class.

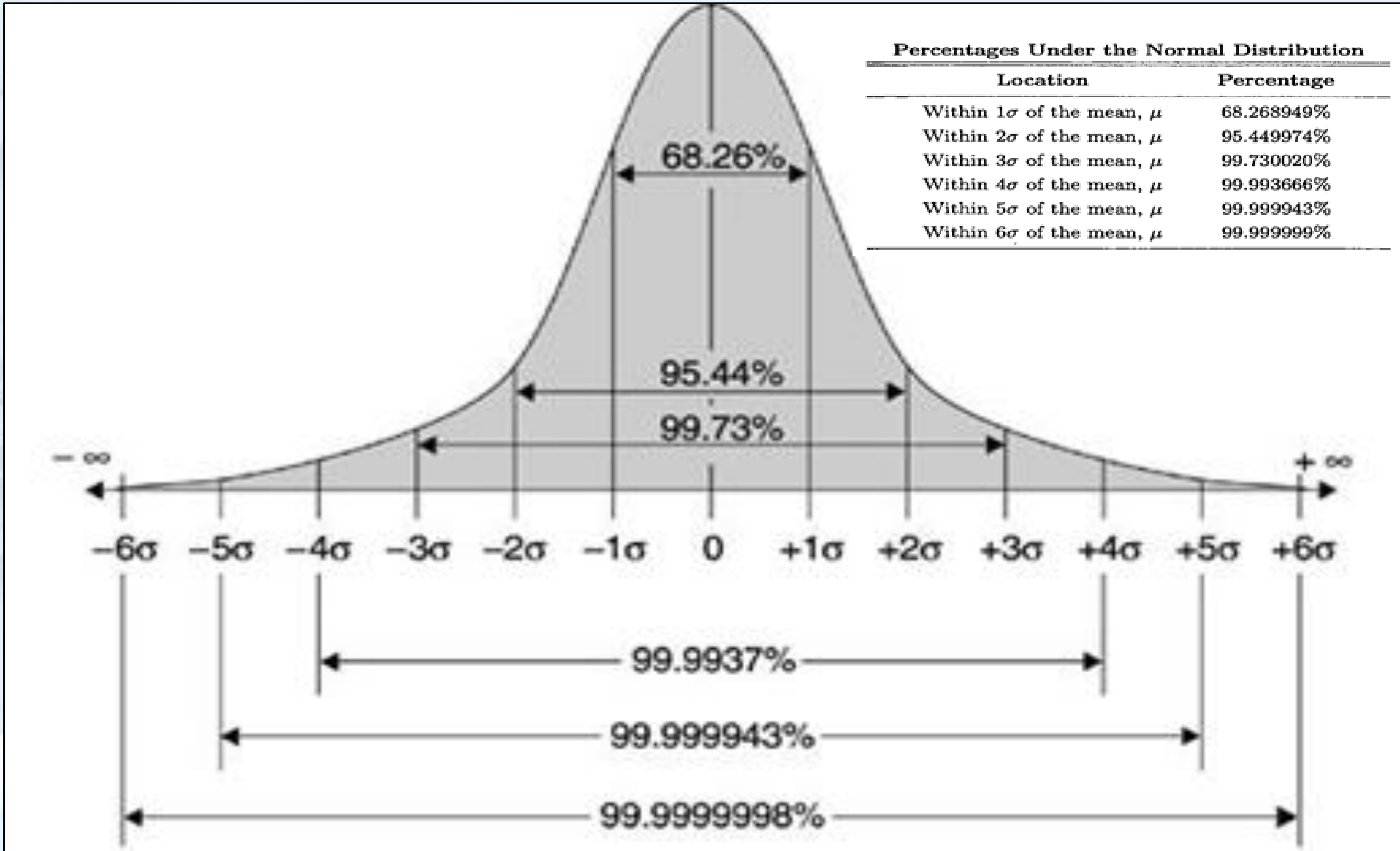


THE NORMAL DISTRIBUTION

A Normal Process

- Data from a Normal Process will produce a continuous, symmetrical, bell-shaped curve.
  - 68.26% of the individuals will be within plus or minus one standard deviation of the mean
  - 95.44% and within two standard deviations
  - 99.73% within three standard deviations
- These percentages are the basis for control charts, limits, analysis and capability decisions.

Normal Distribution



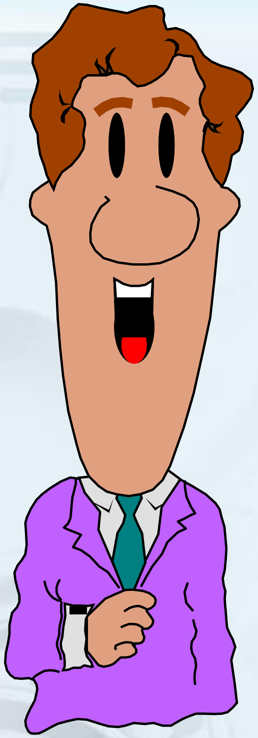
Percentages Under the Normal Distribution

Location	Percentage
Within 1σ of the mean, μ	68.268949%
Within 2σ of the mean, μ	95.449974%
Within 3σ of the mean, μ	99.730020%
Within 4σ of the mean, μ	99.993666%
Within 5σ of the mean, μ	99.999943%
Within 6σ of the mean, μ	99.999999%

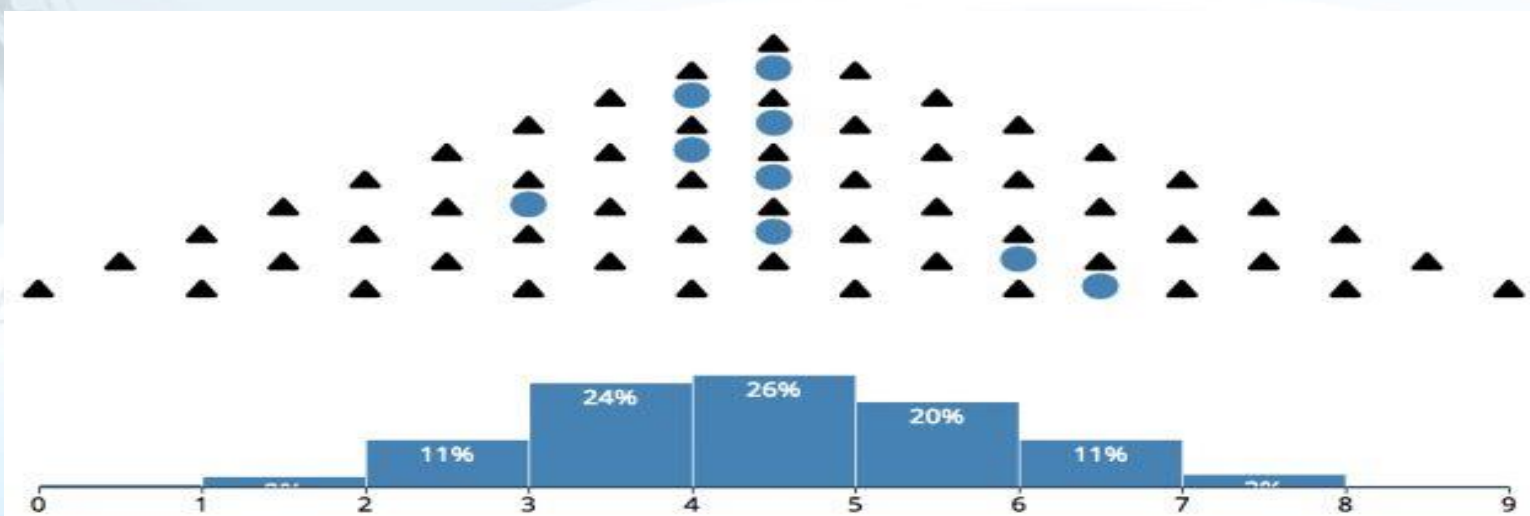
The Central Limit Theorem

- The most important application of the normal distribution in statistical process control is the fact that *in many circumstances, the distribution of the sample mean, $\overline{X}$, is normal.*

- However, if the base distribution is not normal, we can still take advantage of Normal Theory by the Central Limit Theorem.

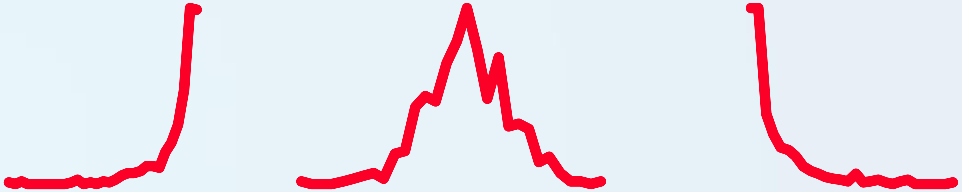


The form of the distribution of sample means approaches the form of a normal distribution as the size of the sample increases.

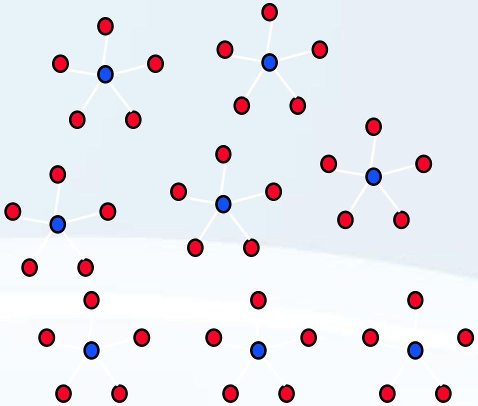


What This Means is That...

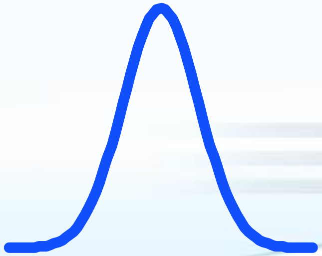
If I have a group of data, which
its distribution shape is any form:



And you create subgroups out of that data:



The distribution of the **averages** of those subgroups will always be a **narrower and more normally shaped distribution.**



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Normal Theory and the Central Limit Theorem

- The Central Limit Theorem is one of the most important concepts in all of statistics.
- This indicates that it is almost always the case that the shape of the distribution of values of $\overline{X}$ is either exactly normal or approximately normal.
- Non-normal distributions such as *flat* or *uniform*, *exponential*, or *skewed*, benefit from the Central Limit Theorem applications in almost all process control applications in manufacturing.

Chapter 1: SPC Background – What We Covered

Learning Objectives

You should now be able to describe:

- The voice of the customer and the voice of the process
- The purpose of SPC
- The difference between prevention and detection
- Process control and process capability
- Over-control
- Location statistics and spread statistics

Chapter Agenda

- Process Fundamentals
- Prevention vs. Detection
- Process Variation
- Control vs. Capability
- Basic Statistics
- **Breakout Exercise 1**
- Normal Distribution
- Central Limit Theorem

Chapter 2: Introduction to Control Charts – What We Will Cover

Learning Objectives

At the end of this chapter, you will be able to describe:

- What statistical control means
- Different types of control charts
- The elements of control charts

Chapter Agenda

- Statistical Control
- Types of Control Charts
- Basic Control Chart Elements

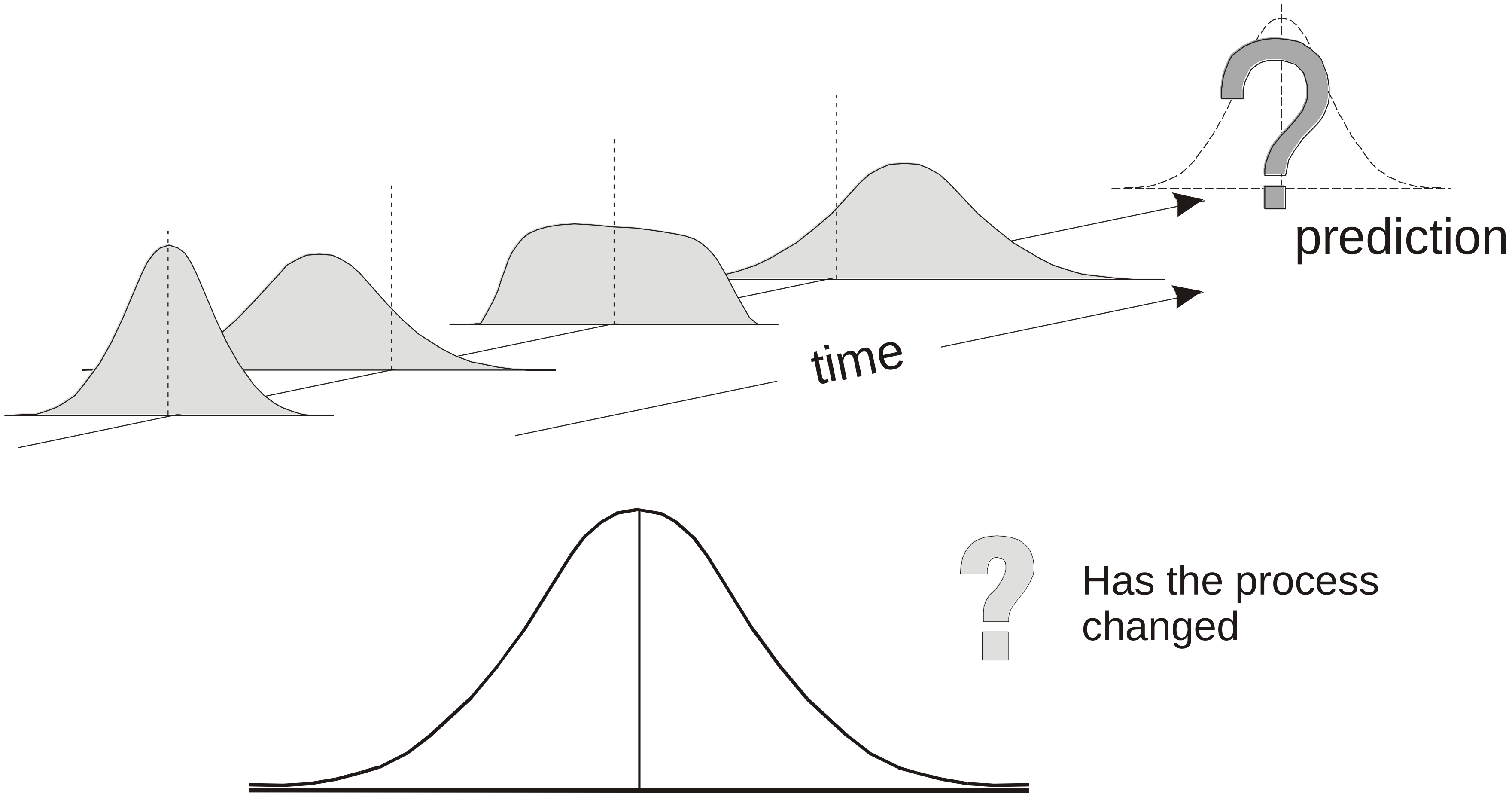
STATISTICAL CONTROL

Performance Evaluation – IATF 16949

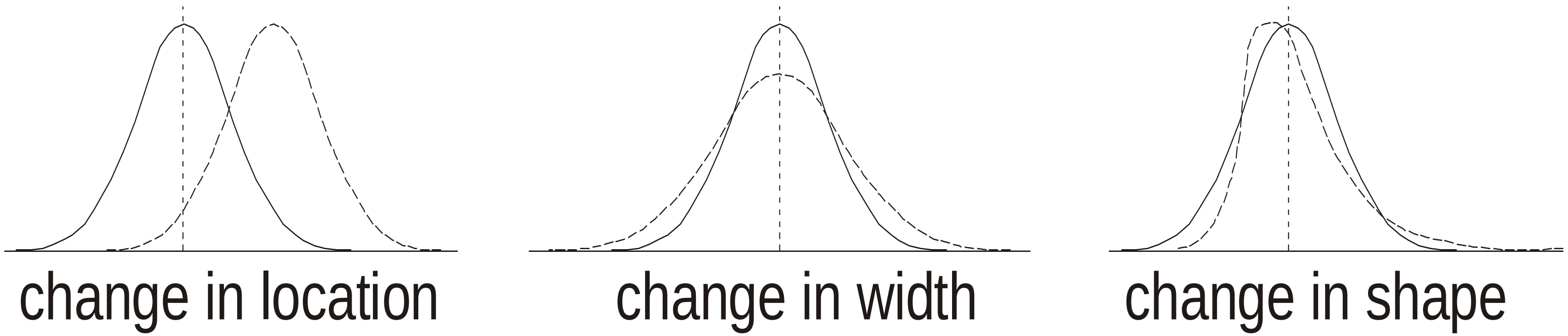
The following clauses from IATF 16949:2016 reference monitoring, measurement and evaluation:

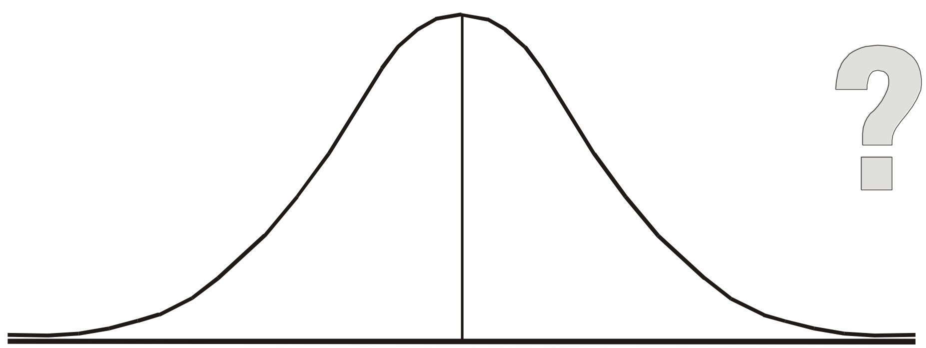
- 9.1.1.1 Monitoring and Measurement of Manufacturing Processes
- 9.1.1.2 Identification of Statistical Tools
- 9.1.1.3 Application of Statistical Concepts

Is the Process in Statistical Control?



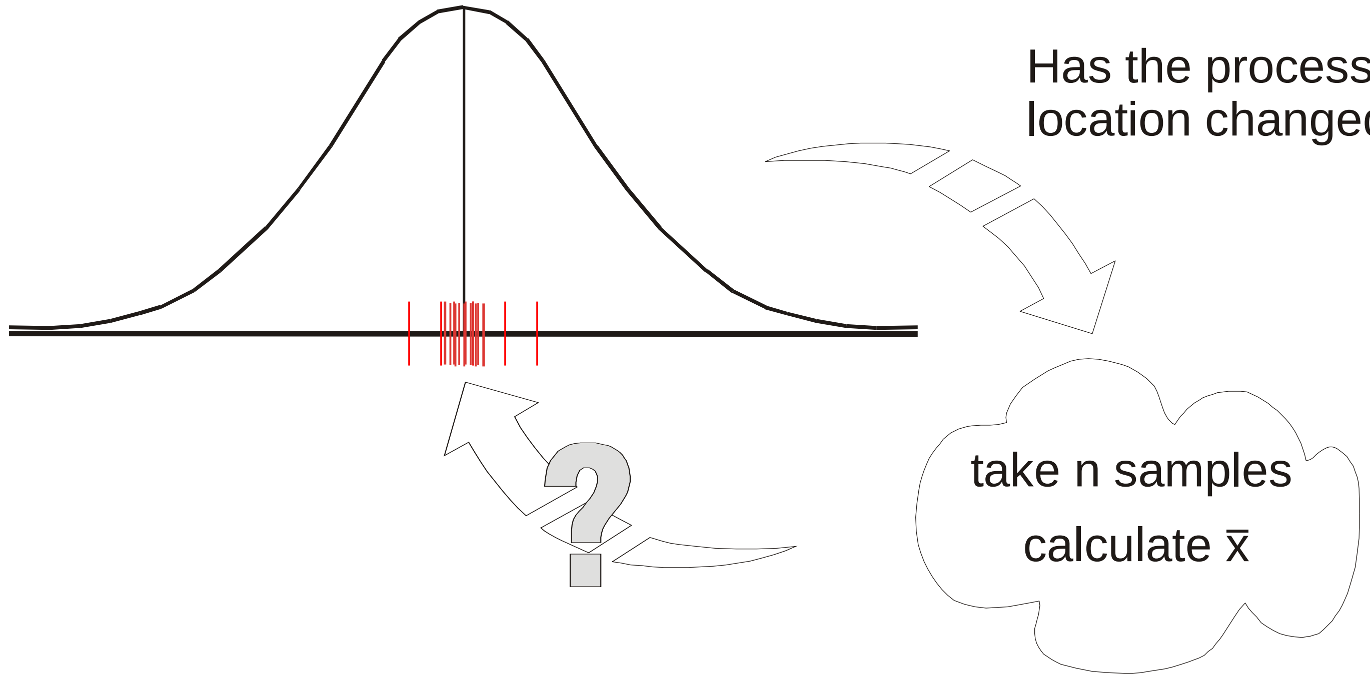
Is the Process in Statistical Control?



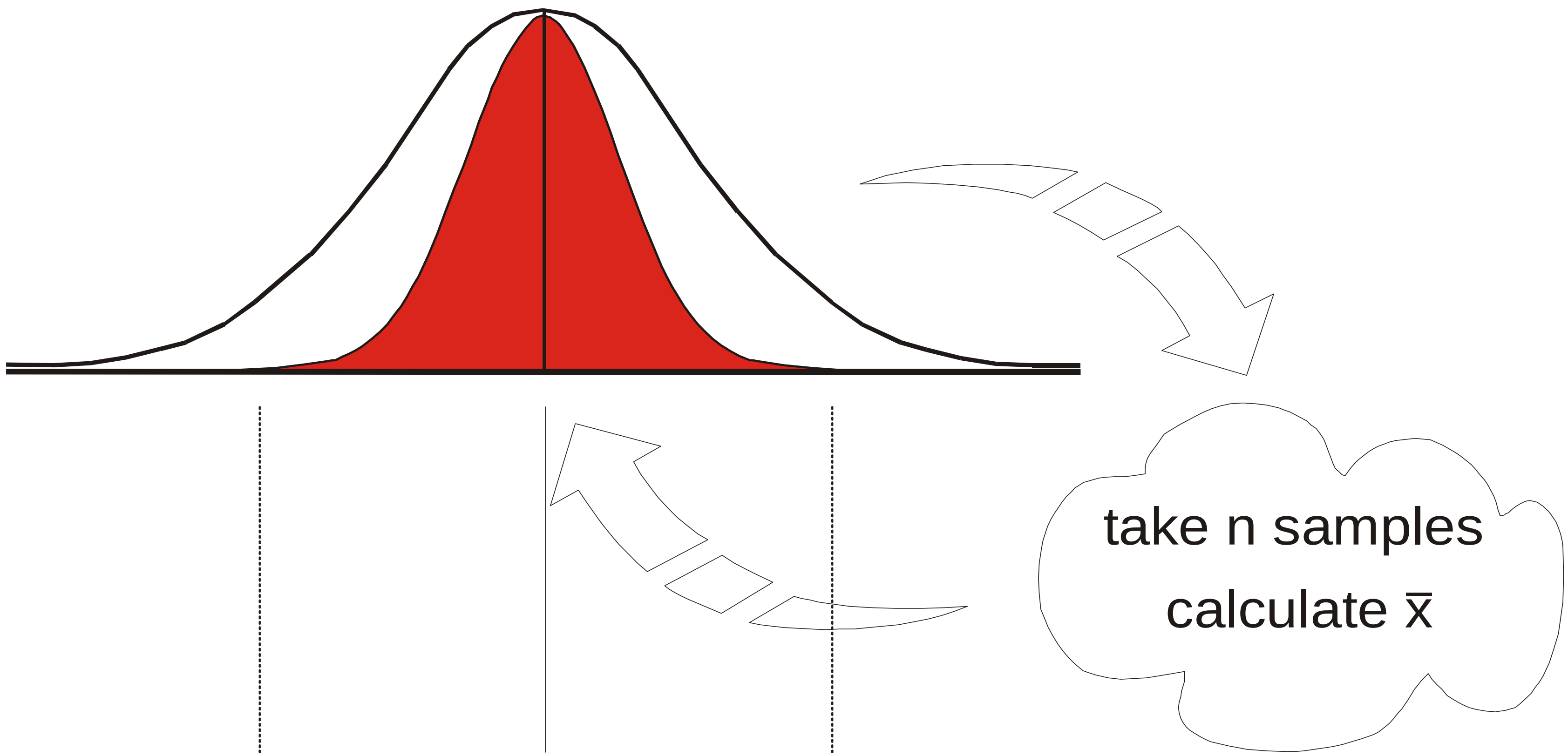


Has the process
changed

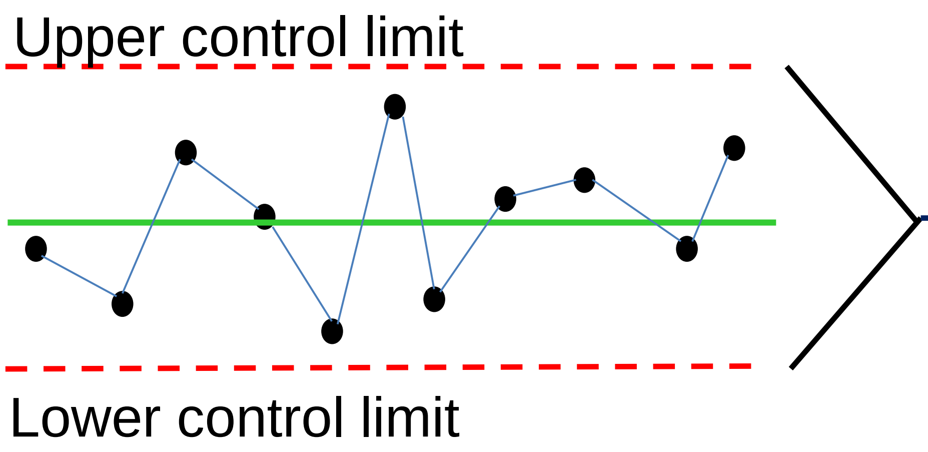
Is the Process in Statistical Control?



Is the Process in Statistical Control?



Due to Sampling Variation



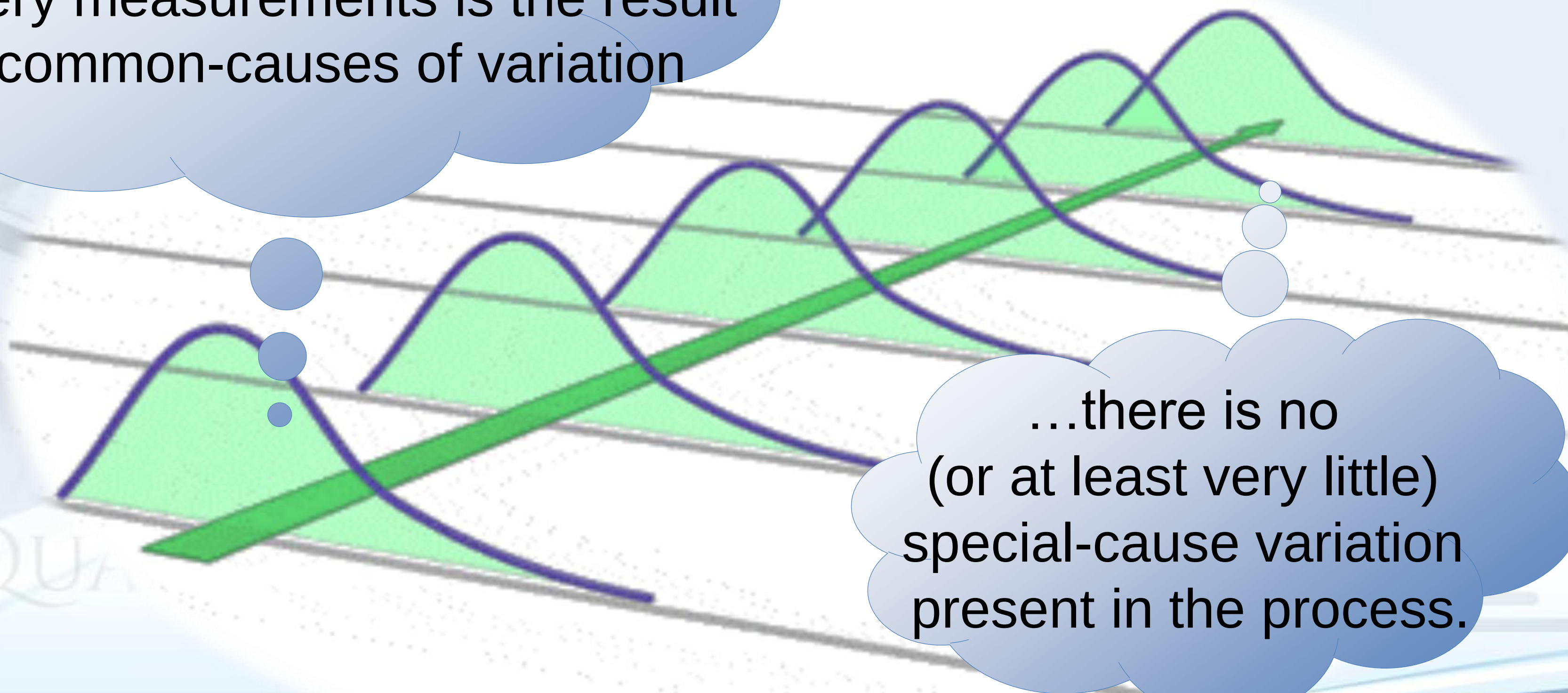
Control Limits show how much ***random*** variation exists in the sampling plan.

Due to Common Causes

Process Control

- When a process is in (statistical) control

...all variation in all important process, product, or service delivery measurements is the result of common-causes of variation



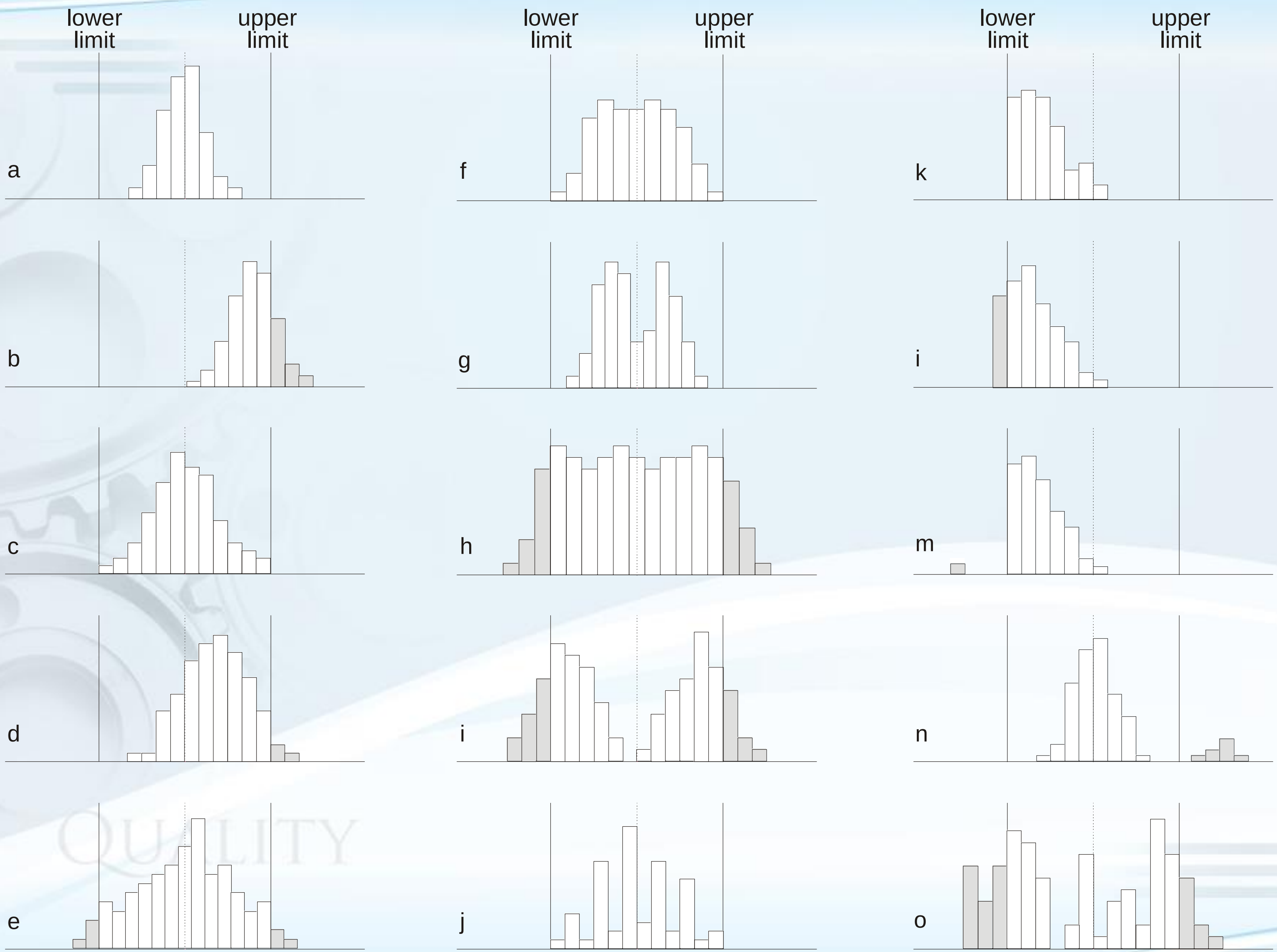
…there is no (or at least very little) special-cause variation present in the process.

The Only Way to Prove a Process is in Statistical Control is by Control Charts

A Diversion

- Although control charts are the only way to verify a process is in statistical control, out of control processes can be identified using other statistical tools: e.g., histograms.
- On the following pages identify which processes are out of statistical control.

Remember: special causes = changes



Examples of Process Variation

Control Charts

- If we are going to watch for patterns or other indicators of control, we should have some control limits. Because the normal process is useful in this situation, we have some formulas that can help us and a mathematical table to aid with calculations.

TYPES OF CONTROL CHARTS

Types of Variable Charts

Basic Charts

- Average and Range Charts ($\overline{X}$ & R charts)
  - When data is readily available
- Average and Standard Deviation Charts ($\overline{X}$ & S charts)
  - When sigma is readily available and the subgroup size is ten or more
- Individual with Moving Range Chart (X & MR charts)

Other Charts

- Moving Average and Moving Range Chart (MX & MR charts)
- EWMA Charts
- CUSUM Charts
- Median and Range Charts
- Run Charts – Note: not a Control Chart

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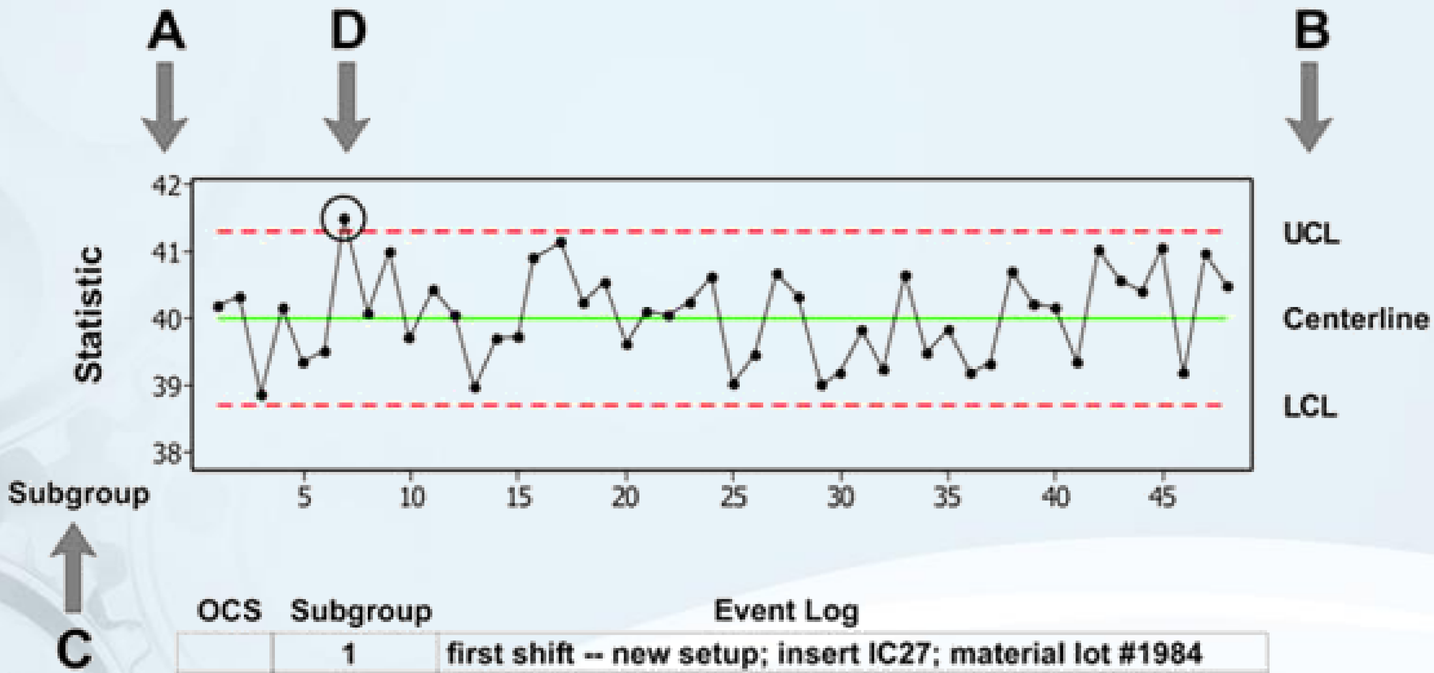
BASIC CONTROL CHART ELEMENTS

Basic Chart Elements

- There is no single “approved” manner of displaying control charts; however the reasons for the use of control charts must be kept in mind.

- Any format is acceptable as long as it contains the following:
  A. Appropriate Scale
  B. UCL, LCL and Centerline
  C. Subgroup Sequence / Timeline
  D. Identification of Out-of-Control Plotted Values
  E. Event Log

Basic Chart Elements



OCS	Subgroup	Event Log
	1	first shift -- new setup; insert IC27; material lot #1984
	4	second shift -- new setup; insert IC27; material lot #1931
	6	third shift -- new setup; insert IC84; material lot #1QR50
***	7	first shift -- material lot #2179, replaced bad insert
	9	third shift -- material lot #2193
	12	first shift -- material lot #1950
	23	second shift -- material lot #ZM18
	etc.	

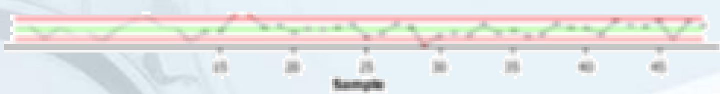
E

Basic Chart Elements

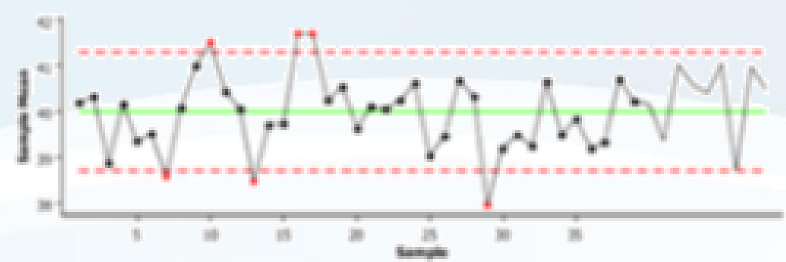
A. Appropriate Scale

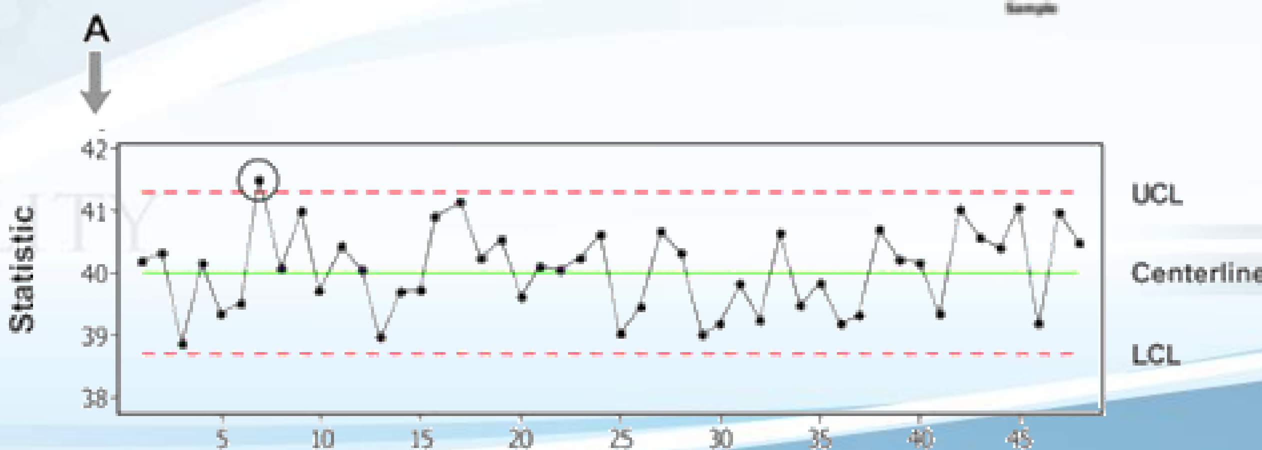
- The scale should be such that the natural variation of the process can be easily viewed.
- A scale which yields a "narrow" control chart does not enable analysis and control of the process.

NO



YES

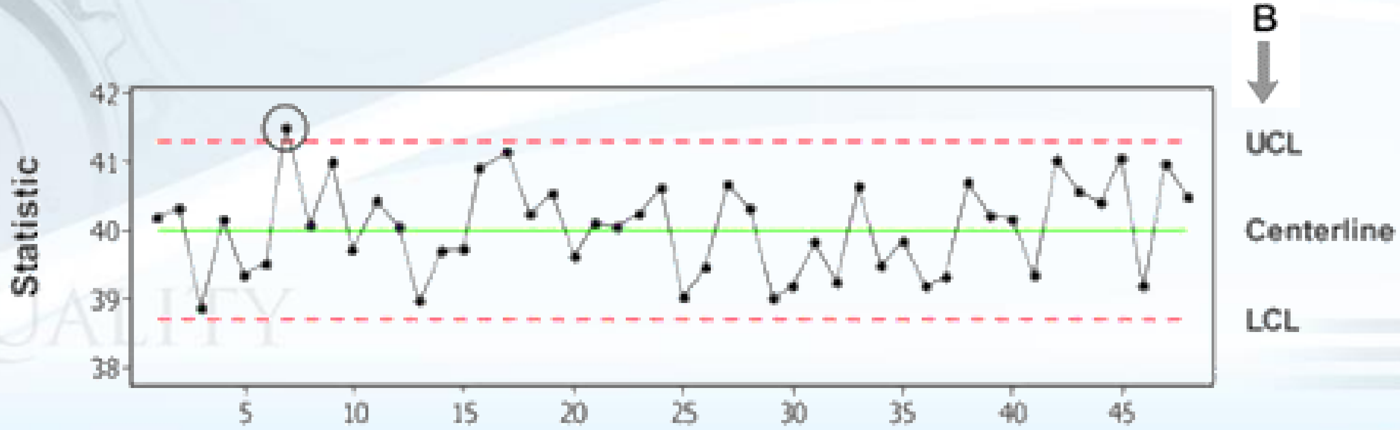




Basic Chart Elements

B. UCL, LCL and Centerline

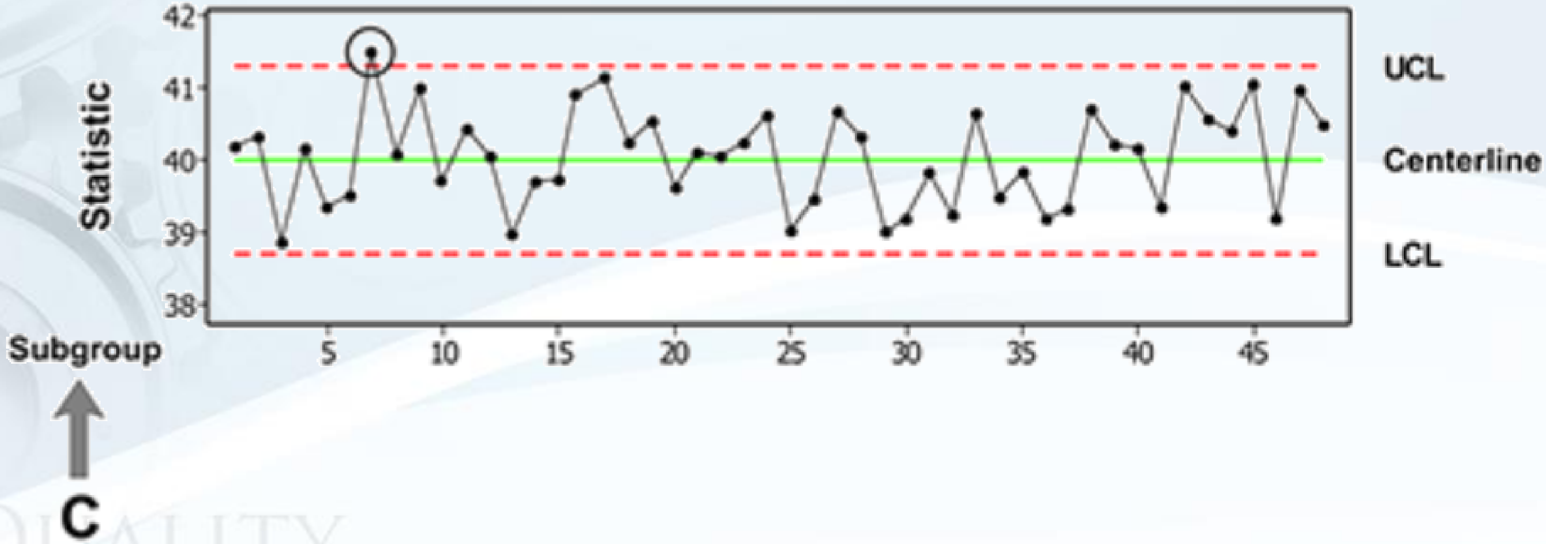
- The ability to determine outliers which signal special causes the control chart requires control limits based on the sampling distribution.
- Specifications limits should not be used in place of valid control limits for process analysis and control.
- The control chart requires a centerline based on the sampling distribution in order to allow the determination of non-random patterns which signal special causes.



Basic Chart Elements

C. Subgroup Sequence / Timeline

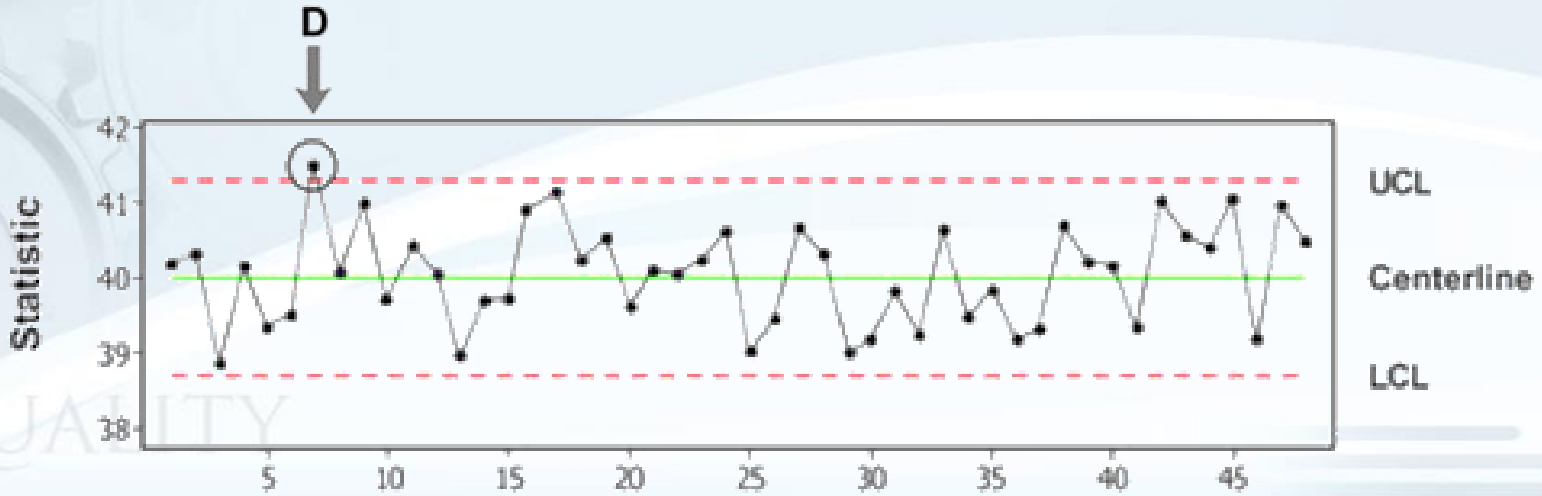
– Maintaining the sequence in which the data (subgroups) are collected provides indications of "when" a special cause occurs and whether that special cause is time-oriented. O$M$$NEX



Basic Chart Elements

D. Identification of Out-of-Control Plotted Values

- Plotted points which are out of statistical control should be identified on the control chart. O$M$$NEX
- For process control, the analysis for special causes and their identification should occur as each sample is plotted as well as periodic reviews of the control chart as a whole for non-random patterns.



Basic Chart Elements

E. Event Log

- Besides the collection, charting, and analysis of data, additional supporting information should be collected.
- This information should include any potential sources of variation as well as any actions taken to resolve *out-of-control signals* (OCS).
- This information can be recorded on the control chart or on a separate event log.
- If there has not been any change in the process between subgroups, it is not necessary to include an entry on the process event log.

OCS	Subgroup	Event Log
	1	first shift -- new setup; insert IC27; material lot #1984
	4	second shift -- new setup; insert IC27; material lot #1931
	6	third shift -- new setup; insert IC84; material lot #1QR50
•••	7	first shift -- material lot #2179, replaced bad insert
	9	third shift -- material lot #2193
	12	first shift -- material lot #1950
	23	second shift -- material lot #ZM18
	etc.	

E

Basic Chart Elements

- For control charts which are included as a part of a report and for those which are maintained manually the following “header” information should be included:
  - **What:** part/product/service name and number/identification
  - **Where:** operation/process step information, name/identification
  - **Who:** operator and appraiser
  - **How:** measurement system used, name/number, units (scale)
  - **How Many:** subgroup size, uniform or by sample
  - **When:** sampling scheme (frequency and time)

Chapter 2: Introduction to Control Charts – What We Covered

Learning Objectives

You should now be able to describe:

- What statistical control means
- Different types of control charts
- The elements of control charts

Chapter Agenda

- Statistical Control
- Types of Control Charts
- Basic Control Chart Elements

Chapter 3

Basic Variable Control Charts

Chapter 3: Basic Variable Control Charts – What We Will Cover

Learning Objectives

At the end of this chapter, you will be able to describe:

- How to construct an $\overline{X}$ & R chart
- The type of variation seen on the $\overline{X}$ chart
- What the spread of control limits on the $\overline{X}$ chart represent
- The type of variation seen on the range chart
- Applications for an X & MR chart
- Differences between an $\overline{X}$ & R chart and the X & MR chart

Chapter Agenda

- $\overline{X}$ & R
- **Breakout Exercise 2**
- **Breakout Exercise 3**
- X & MR
- **Breakout Exercise 4**

Basic Control Charts

Variables Charts:

- Average and Range Charts
- Average and Sigma Charts
- Individual and Moving Range Charts
- *Median and Range Charts*

Attributes Charts:

- *p*, *np*, *c*, and *u* Charts

$\overline{X}$ & R Chart

- When we have variable data
- There is a need to establish process variation
- When we can obtain samples of constant size between 2-10 consecutive pieces
- When pieces are produced under similar conditions with a short interval between production of pieces

$\overline{X}$ & R Chart

To calculate the control limits for the chart we use the following formulas:

Subgroup Average:

$$\overline{X} = \frac{x_1 + x_2 + \ldots + x_n}{n};$$

$n =$ number of samples in a subgroup

Subgroup Range:

$R = x_{Max} - x_{Min}$ (within each subgroup)

Grand Average:

$$\overline{\overline{X}} = \frac{\overline{X}_1 + \overline{X}_2 + \ldots + \overline{X}_k}{k};$$

$k =$ number of subgroups used to determine the Grand Average and Average Range

Average Range:

$$\overline{R} = \frac{R_1 + R_2 + \ldots R_k}{k};$$

Centerline	**Control Limits**	
$CL_{\overline{X}} = \overline{\overline{X}}$	$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R}$	$LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R}$
$CL_R = \overline{R}$	$UCL_R = D_4\overline{R}$	$LCL_R = D_3\overline{R}$

n	2	3	4	5	6	7	8	9	10
D4	3.27	2.57	2.28	2.11	2	1.92	1.86	1.82	1.78
D3	-	-	-	-	-	0.08	0.14	0.18	0.22
A2	1.88	1.02	0.73	0.58	0.48	0.42	0.37	0.34	0.31

Process for $\overline{X}$ & R Chart

1. Determine sample size, typically between 2-10 pieces
   - For 10 or more sample sizes, use an $\overline{X}$ & S chart for more sensitivity
2. Establish the frequency of taking measurements
3. Collect data and establish control limits
4. Calculate average and record for each sample
5. Determine the range and record for each sample
6. Plot the average and range on chart
7. Use the chart to monitor the process

Breakout Exercise 2

Plotting Data

QUALITY

Breakout Exercise 2: Plotting Data

Instructions

- Turn to Exercise 2 in your workbook.
- Plot the data from Exercise 1 on the $\overline{X}$ & R chart.
- Include the $\overline{\overline{X}}$ & $\overline{R}$ (averages) or center lines.
- Analyze at this point.

We will introduce some rules designed to detect certain kinds of non-random patterns.

Before presenting the rules, however, try to detect the presence of **special cause(s)** of variation in the process whose control chart you started.

Conduct an analysis of the chart without regard to any specific rules that have been previously learned.

Answer: $\overline{X}$ & R Chart

Breakout Exercise 3

Control Charts

Breakout Exercise 3: Control Charts

Instructions

- Turn to Exercise 3 in your workbook.
- Calculate the control limits for your chart using the formulas provided and the included Control Chart Constants Matrix.

X & MR Chart (or I & MR Chart)

- Individual reading and a moving range are useful for short runs and for parts tested by destructive testing.
- Also called I & MR charts – substituting I for individual in place of X.
- Control limits are calculated from the average moving range, similar to the $\overline{X}$ & R chart.
- Chart does ***not*** separate piece-to-piece variability.
- Chart is not as sensitive to changes in the process as the $\overline{X}$ & R chart.

X & MR Chart (or I & MR Chart)

To calculate the control limits for the chart we use the following formulas:

Individual Value: $x_i,\ i = 1,\ldots,k$ individual values :

Average of Individual Values:

$$\bar{X} = \frac{x_1 + x_2 + \ldots x_k}{k}$$

Moving Range:

$$MR_i = |x_i - x_{i-1}|,\ i = 2\ldots k$$

(Range between current value and previous value.)

Average Moving Range:

$$\overline{MR} = \frac{MR_2 + MR_3 + \ldots MR_k}{k-1}$$

Estimate of the Standard Deviation of X :

$$\hat{\sigma}_C = \bar{R} / d_2$$

Centerline	**Control Limits**	
$CL_X = \bar{X}$	$UCL_X = \bar{X} + E_2\bar{R}$	$LCL_X = \bar{X} - E_2\bar{R}$
$CL_R = \bar{R}$	$UCL_R = D_4\bar{R}$	$LCL_R = D_3\bar{R}$

n	2	3	4	5	6	7	8	9	10
D4	3.27	2.57	2.28	2.11	2	1.92	1.86	1.82	1.78
D3	-	-	-	-	-	0.08	0.14	0.18	0.22
E2	2.66	1.77	1.46	1.29	1.18	1.11	1.05	1.01	0.98

OMNEX

Breakout Exercise 4

X & MR Charts

Breakout Exercise 4: X & MR Charts

Instructions

- Turn to Exercise 4 in your workbook.
- Look at the enclosed chart and identify the differences and similarities to the $\overline{X}$ & R charts.
- Analyze the viscosity chart for capability and control.

Answer: X & MR Chart (or I & MR Chart)

Answer

Chapter 3: Basic Variable Control Charts – What We Covered

Learning Objectives

You should now be able to describe:

- How to construct an $\overline{X}$ & R chart
- The type of variation seen on the $\overline{X}$ chart
- What the spread of control limits on the $\overline{X}$ chart represent
- The type of variation seen on the range chart
- Applications for an X & MR chart
- Differences between an $\overline{X}$ & R chart and the X & MR chart

Chapter Agenda

- $\overline{X}$ & R
- **Breakout Exercise 2**
- **Breakout Exercise 3**
- X & MR
- **Breakout Exercise 4**

OMNEX

Chapter 4

Basic Attribute Control Charts

Chapter 4: Basic Attribute Control Charts – What We Will Cover

Learning Objectives

At the end of this chapter, you will be able to describe:

- Key characteristics of attribute control charts
- The four attribute charts
- When to use attribute charts

Chapter Agenda

- Characteristics of Attribute Charts
- Types of Attribute Charts
  - *np* Chart
  - *p* Chart
  - *c* Chart
  - *u* Chart

QUALITY

OMNEX

Characteristics of Attribute Charts

- Subgroup or sample size needs to be greater than 50.
- Average number of defects/defectives should equal 4 or more.
- If the subgroup size varies by more than 10%, control limits need to be recalculated.
  - With computer generated charts, any variation in *n* should have recalculated limits.
- The defects and defectives plotted in attribute charts are categorized into a Pareto Chart to determine the vital few.
  - Reducing the defects needs a fundamental system change.

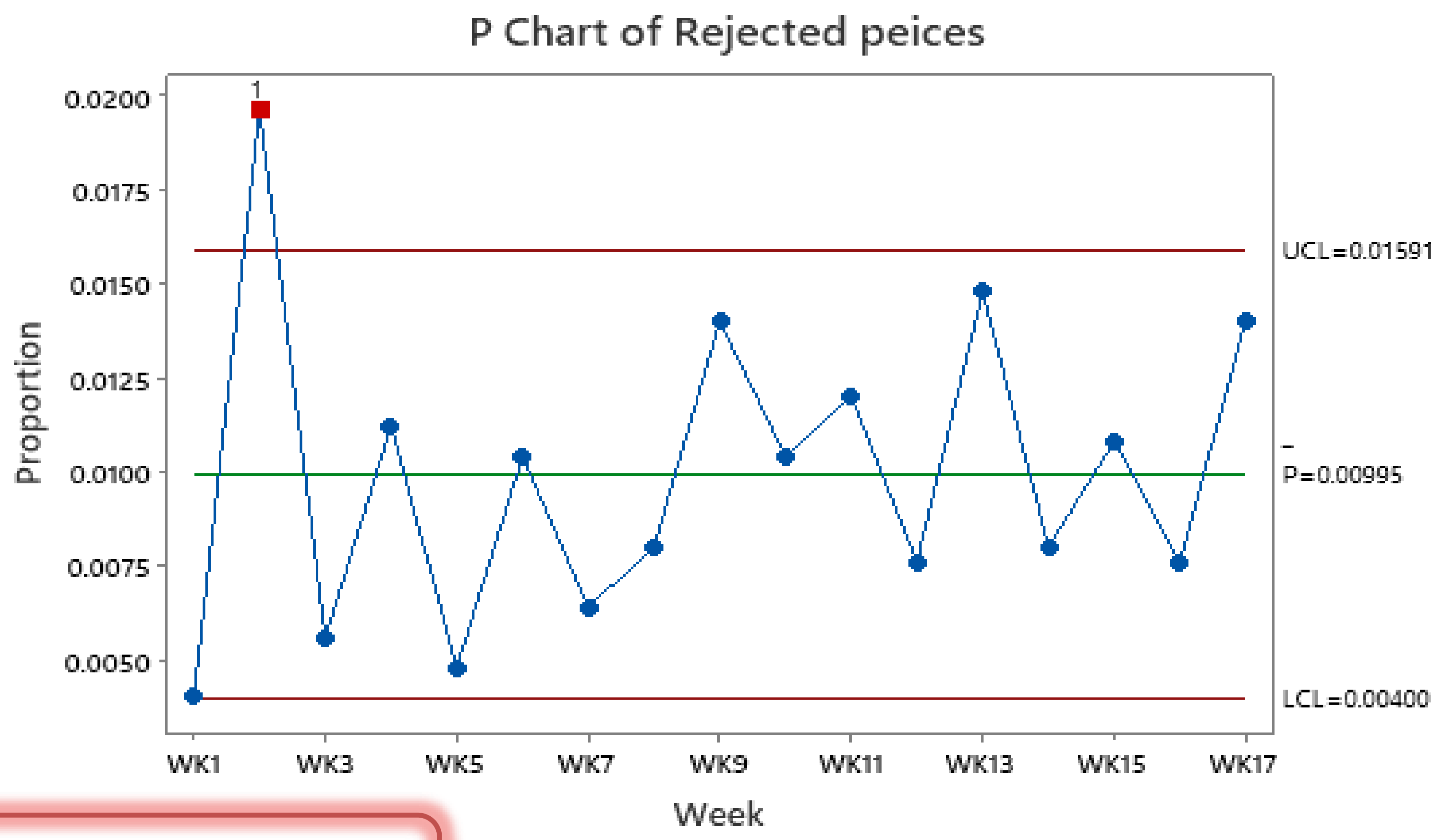
Types of Attribute Charts

- **Proportion Nonconforming (*p* Chart)**
  - For defectives with varying sample sizes
- **Nonconforming Product (*np* Chart)**
  - For defectives with constant sample size
- **Number of Nonconformities (*c* Chart)**
  - For defects with fixed sample size
- **Nonconformities Per Unit (*u* Chart)**
  - For defects with varying sample size

p Chart – Percent / Proportion

- Analyzes percentages or proportions
  - Commonly used with **Percentage of Non-conforming Product**, also called ***Fraction Defective***.
  - Can be used with any process percent not just non-conforming product
- Sample size can be different for individual readings

p Chart: with Constant Sample Size



Constant Sample Size of 2500

Minitab 19

p Chart

To analyze the proportion of defectives or nonconforming use the following formulas for the chart limits:

Guideline:

Since the control limits are based on a normal approximation, the sample size used should be such that $n\bar{p} \geq 5$.

Individual Value

$$p_i = \frac{np_i}{n_i}$$

n_i = number of parts inspected;

np_i = number of nonconforming items found

Average of Individual Values

$$\bar{p} = \frac{np_1 + np_2 + ... + np_k}{n_1 + n_2 + ... + n_k}$$

where k = number of subgroups

$$\bar{p} = \frac{p_1 + p_2 + ... + p_k}{k}$$

if all the n_i's are equal

p Chart

To analyze the proportion of defectives or nonconforming use the following formulas for the chart limits:

If the sample size is constant (n)

Control Limits

$$UCL_p = \overline{p} + 3\frac{\sqrt{\overline{p}(1-\overline{p})}}{\sqrt{n}}$$

$$LCL_p = \overline{p} - 3\frac{\sqrt{\overline{p}(1-\overline{p})}}{\sqrt{n}}$$

Constant control limits when the sample size varies

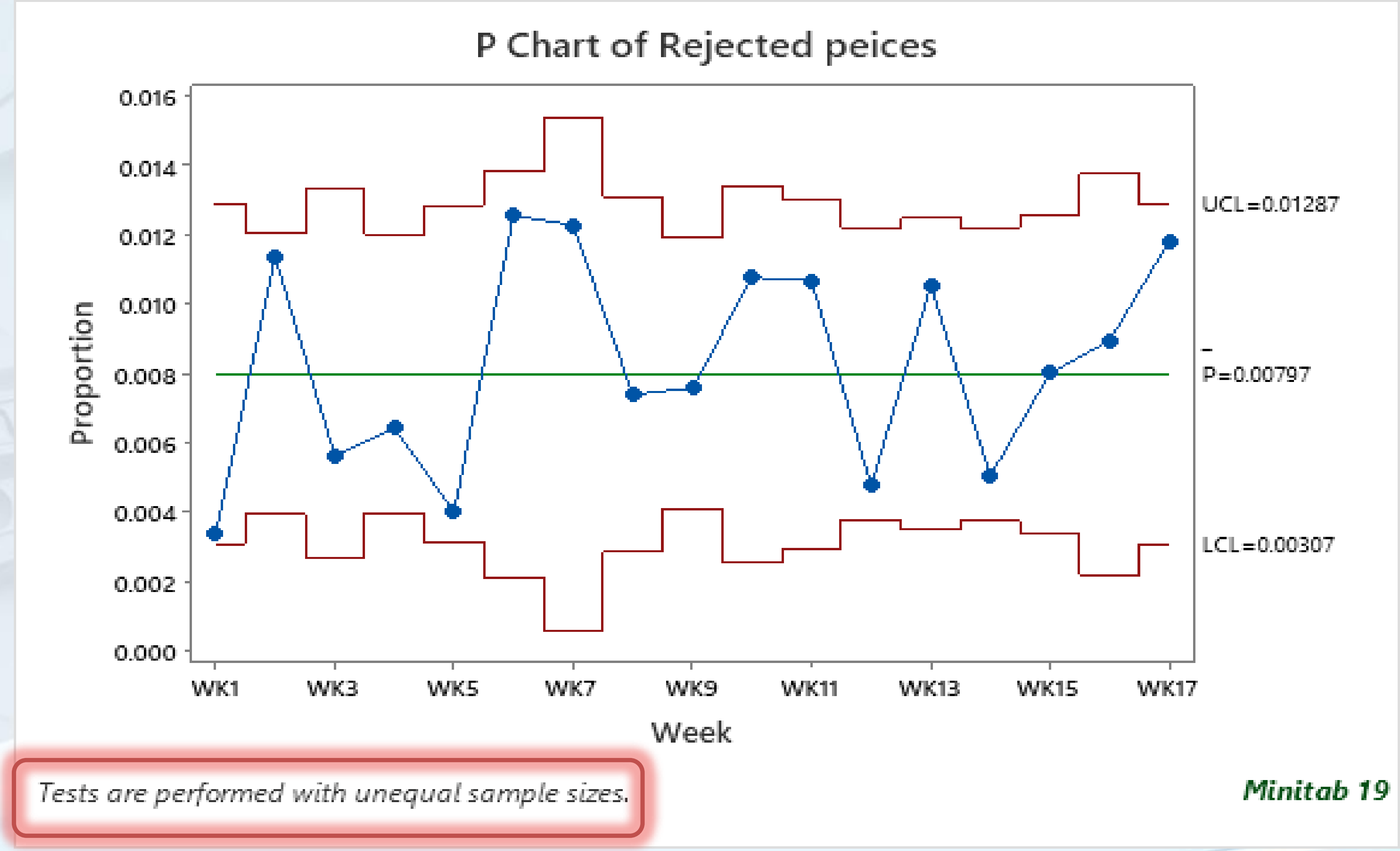
(for situations where $\frac{\min n_i}{\max n_i} \geq 0.75$)

Control Limits

$$UCL_p = \overline{p} + 3\frac{\sqrt{\overline{p}(1-\overline{p})}}{\sqrt{\overline{n}}}$$ ($\overline{n}$ = average sample size)

$$LCL_p = \overline{p} - 3\frac{\sqrt{\overline{p}(1-\overline{p})}}{\sqrt{\overline{n}}}$$ ($\overline{n}$ = average sample size)

p Chart



Notice how the control limits vary according to the sample size

Number of Defectives – *np* Chart

- Total average defectives per lot is the np value, which is the total defectives divided by the number of lots.
- **Sample size cannot be different for individual readings.**
- This chart plots the **COUNT** of the defectives.

np Chart

To calculate the np chart limits, use the following formulas:

Restriction:

Requires a constant subgroup size = n

Guideline:

Since the control limits are based on a normal approximation, the sample size used should be such that $\overline{np} \geq 5$

Individual Value:

np_i n = number of parts inspected;

np = number of nonconforming items found

Average of Individual Values:

$$\overline{np} = \frac{np_1 + np_2 + ... + np_k}{k}$$

OMNEX

np Chart

To calculate the np chart limits, use the following formulas:

Centerline

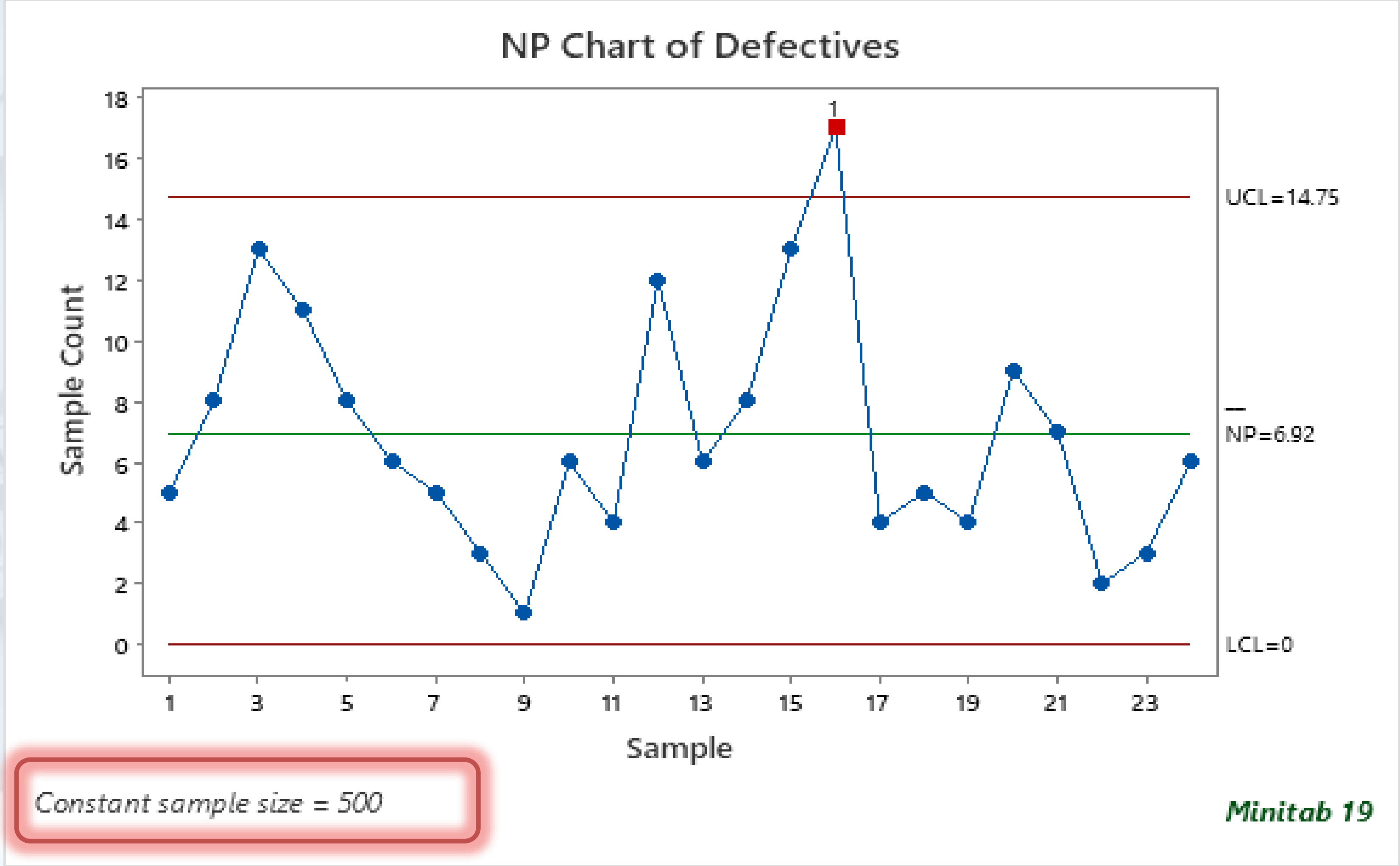
$$CL_{np} = \overline{np}$$

Control Limits:

$$UCL_{np} = \overline{np} + 3\sqrt{\overline{np}(1 - \frac{\overline{np}}{n})} = \overline{np} + 3\sqrt{\overline{np}(1 - \overline{p})}$$

$$LCL_{np} = \overline{np} - 3\sqrt{\overline{np}(1 - \frac{\overline{np}}{n})} = \overline{np} - 3\sqrt{\overline{np}(1 - \overline{p})}$$

np Chart



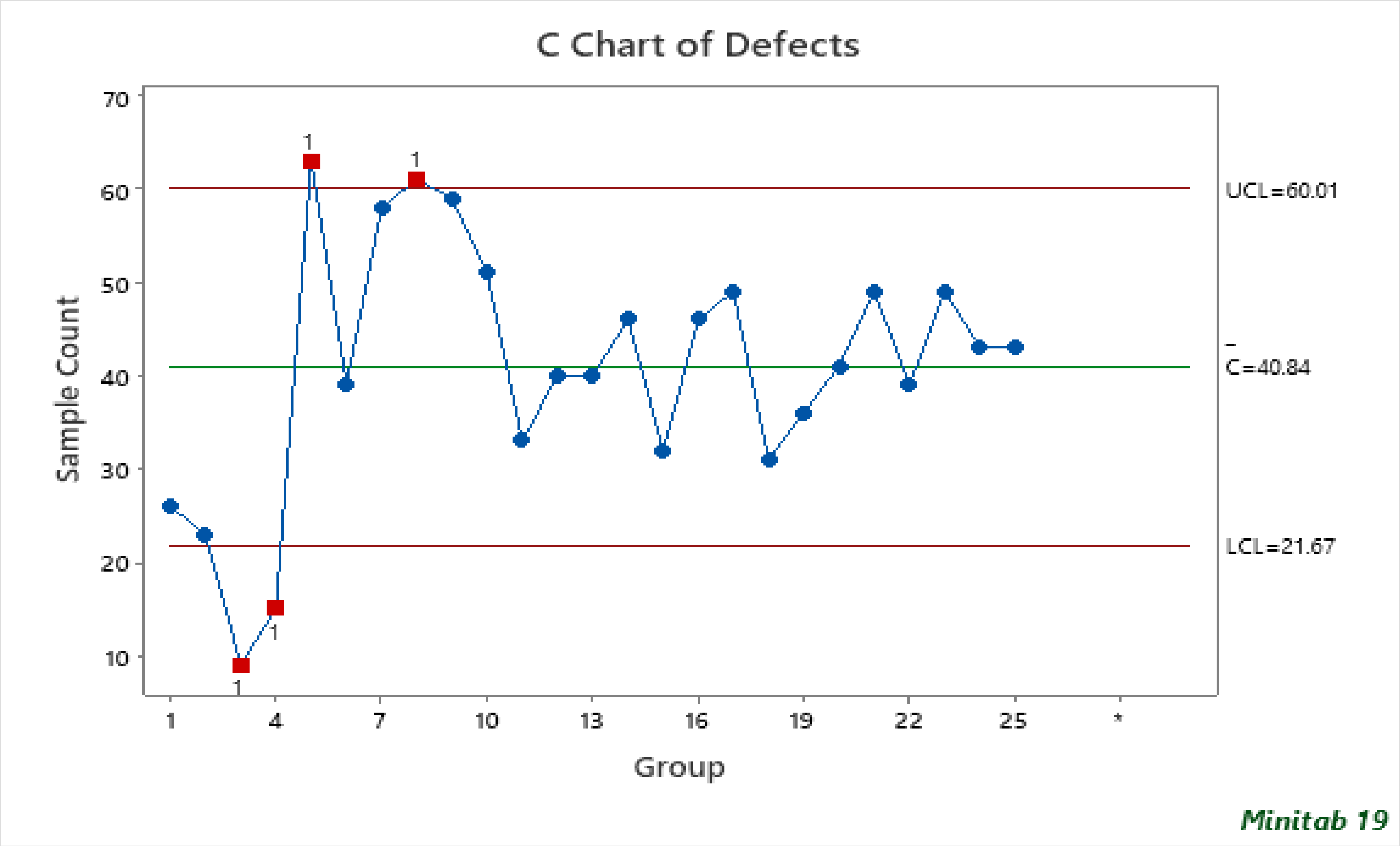
With constant sample size: in this instance 500 units

c Chart

- Counted Attribute data – e.g., Nonconformities count per unit
- Nonconformities are distributed throughout a product, e.g.:
  - Number of defects on a painted part
  - Number of flaws in an assembly operation
- **Effectively a sample (subgroup) size = 1**

When nonconformities can be found from multiple sources or attributed to multiple sources

c Chart



With a constant sample size of “1”

c Chart

To calculate number of defects and the chart limits, use the following formulas:

Restriction:

Requires a constant subgroup size = n

Guideline:

Since the control limits are based on a normal approximation, the sample used must be large enough so that the number of subgroups with $c = 0$ is small

Individual Value:

c_i = number of nonconformities found in sample; $i = 1, \ldots, k$

Average of Individual Values:

$$\bar{c} = \frac{c_1 + c_2 + \ldots + c_k}{k}$$ k = number of samples

Chart Features:

Centerline

$$CL_C = \bar{c}$$

Control Limits

$$UCL_C = \bar{c} + 3\sqrt{\bar{c}}$$

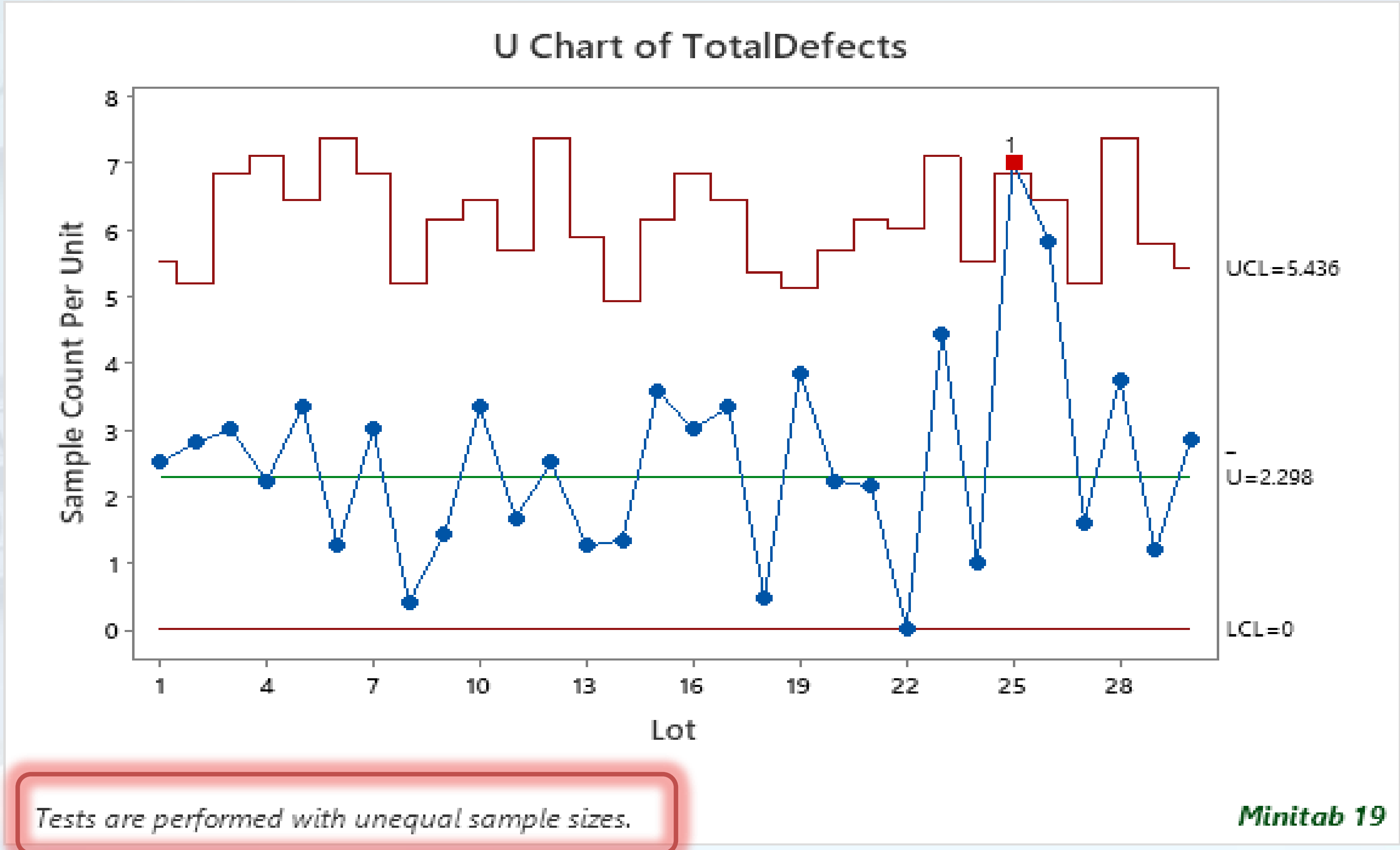
$$LCL_C = \bar{c} - 3\sqrt{\bar{c}}$$

u Chart

- Average number of defects per unit is the *u* value, which is the total defects found in a sample divided by the number of units (pieces) in the sample.
- ***Sample size can be different for individual readings.***

On average, are fewer defects occurring in the assembly process?

u Chart



With unequal / variable sample sizes

u Chart

To calculate the average number of defects per unit or average nonconforming per unit and the chart limits, use the following formulas:

Guideline:

Since the control limits are based on a normal approximation, the sample size used must be large enough so that the number of subgroups with $c = 0$ is small.

Individual Value:

$$u_i = \frac{c_i}{n_i}$$

c_i = number of nonconformities found in sample i;

n_i = is the sample size

Average of Individual Values:

$$\bar{u} = \frac{u_1 + u_2 + \ldots. + u_k}{k}$$

u Chart

To calculate the average number of defects per unit or average nonconforming per unit and the chart limits, use the following formulas:

Centerline

$$CL_u = \bar{u}$$

Control Limits

$$UCL_u = \bar{u} + \frac{3\sqrt{\bar{u}}}{\sqrt{n_i}} = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}}$$

$$LCL_u = \bar{u} - \frac{3\sqrt{\bar{u}}}{\sqrt{n_i}} = \bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}}$$

For constant control limits when the sample size varies

(for situations where $\frac{\min n_i}{\max n_i} \geq 0.75$)

Control Limits:

$$UCL_u = \bar{u} + \frac{3\sqrt{\bar{u}}}{\sqrt{\bar{n}}} = \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}}$$ ($\bar{n}$ = average sample size)

$$LCL_u = \bar{u} - \frac{3\sqrt{\bar{u}}}{\sqrt{\bar{n}}} = \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}}$$ ($\bar{n}$ = average sample size)

Chapter 4: Basic Attribute Control Charts – What We Covered

Learning Objectives

You should now be able to describe:

- Key characteristics of attribute control charts
- The four attribute charts
- When to use attribute charts

Chapter Agenda

- Characteristics of Attribute Charts
- Types of Attribute Charts
  - *np* Chart
  - *p* Chart
  - *c* Chart
  - *u* Chart

Chapter 5

Analyzing Control Charts – Special Cause Criteria

Chapter 5: Analyzing Control Charts – What We Will Cover

Learning Objectives

At the end of this chapter, you will be able to describe:

- At least four of the common out-of-control detection rules
- How to analyze a control chart for out-of-control situations

Chapter Agenda

- Common Out-of-Control Detection Rules
- Patterns that Signal Out-of-Control
- **Breakout Exercise 5**

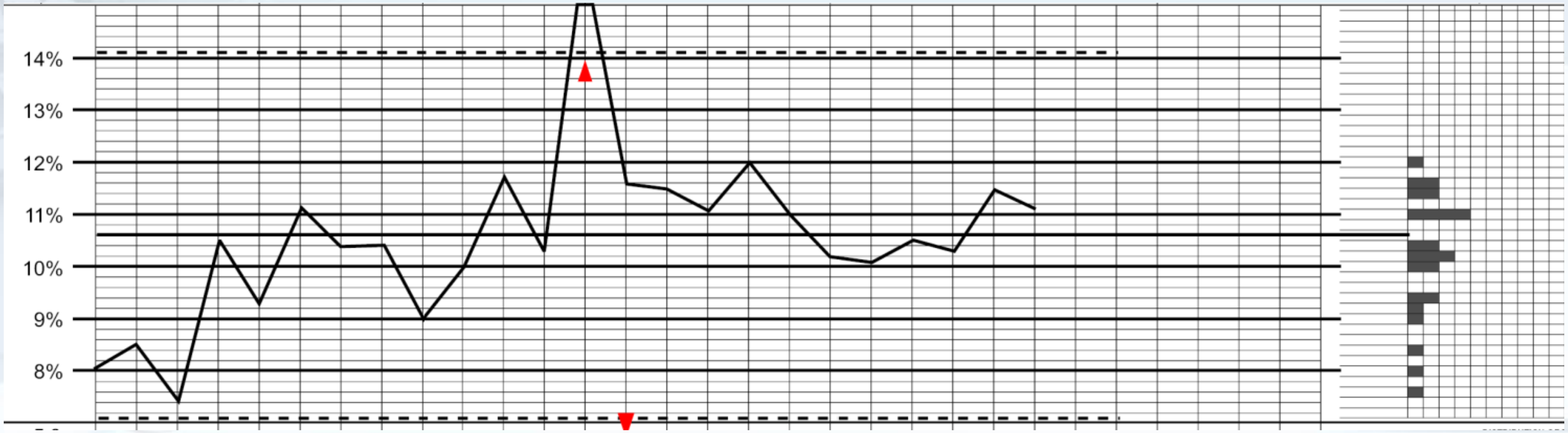
Control Charts

- If a special cause of variation occurs at some point in time, its impact on the statistic will be noticeable.

- The behavior of the statistic will no longer be random; its pattern of values will probably stand out from the common cause of variation.

Special Cause Criteria

- The samples are randomly selected from the population.
- The plotted points will have a normal distribution if the process is in statistical control.



Common Out-of-Control Detection Rules

- A few commonly seen changes in a process are due to…
  - A *gradual change* in the center or spread of the process, possibly due to tool wear, worker fatigue, depletion of a chemical in a solution, changing work conditions, etc.
  - An *abrupt and persistent change* in the center or spread of the process, possibly due to a change in the material or supplier, a change of operators, a change in monitoring procedures, reaction to a new set-up, etc.
  - An *abrupt but temporary change* in the center or spread of the process.
  - *Periodic movements* in the center or spread of the process, possibly due to daily or weekly environmental changes, operator rotations, etc.
  - Other types of *erratic or systematic changes* in the center or spread of the process that experience has shown to be associated with specific causes.

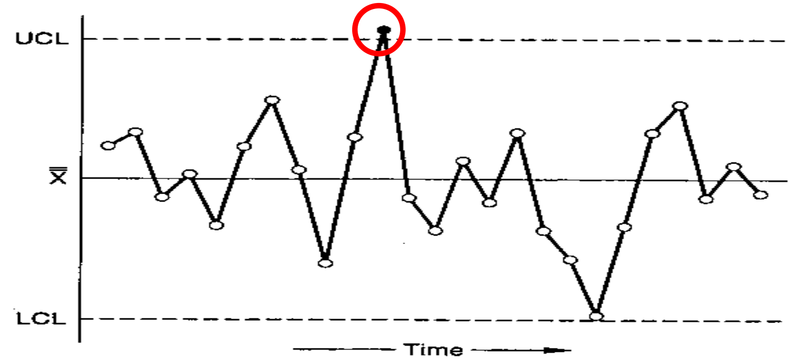
OMNEX

Summary of Typical Special Cause Criteria

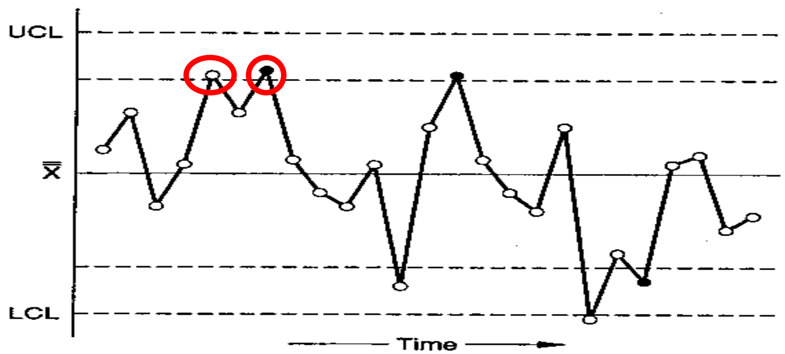
1. One point more than three standard deviations from centerline
2. Seven or more points in a row on same side of centerline
3. Six points in a row, all increasing or all decreasing
4. Fourteen points in a row, alternating up and down
5. Two out of three points > 2 std dev from centerline (same side)
6. Four out of five points > 1 std dev from centerline (same side)
7. Fifteen points in a row within 1 std dev of centerline (either side)
8. Eight points in a row > 1 std dev from centerline (either side)

In this table, "standard deviation" refers to the standard deviation used in the calculations of the control limits.

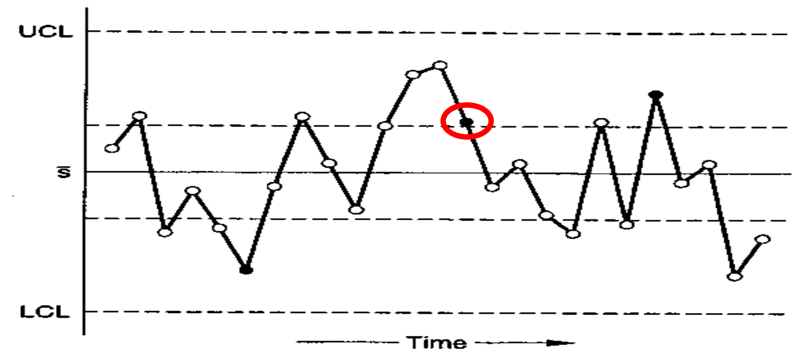
Out of Control Patterns in Control Charts



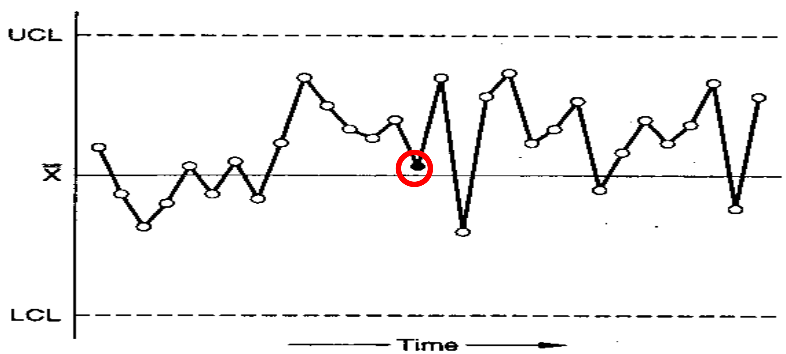
(a) A point outside the control limits.



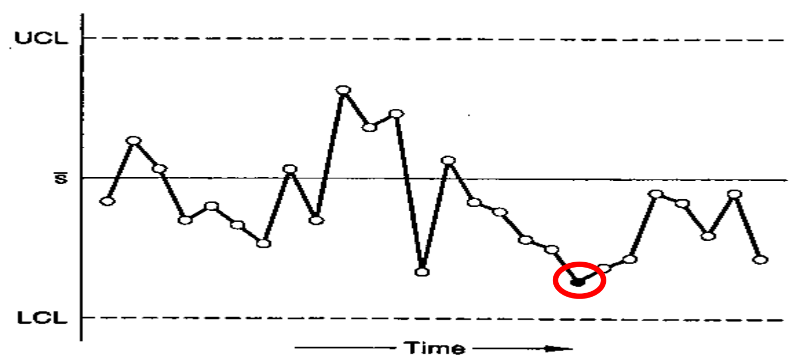
(b) Two out of three points beyond two standard deviations.



(c) Four out of five points beyond one standard deviation.



(d) A run of seven points on one side of the midline.



(e) A trend of length six or "almost trend" of length ten.

Patterns that Signal Out-of-Control

- A few explanations for a point being outside the control limits are...
  - The control limits have been miscalculated.
  - The value of the statistic has been mis-plotted. O*M*N*E*X*
  - The variability of the process has increased or decreased, if the sample is larger than six. (R charts)
  - The center of the process has shifted. ($\overline{X}$ charts)
  - There is a trend in the data. ($\overline{X}$ charts)
  - The measurement system has changed (there is new gaging system or a new inspector).

Summary

- If a control chart includes one of the signals discussed in this segment, it not only indicates that the process is out of control; the nature of the signal also suggests what the root cause of the problem may be.
- The pattern of behavior of its control chart statistic should be completely random.
- The way a control chart signals an out-of-control process is by revealing systematic patterns in the values of the statistic, patterns that have a low probability of occurring when the process is actually in control.

Breakout Exercise 5

Interpreting Control Charts

Breakout Exercise 5: Interpreting Control Charts

Instructions

- Turn to Exercise 5 in your workbook.
- Examine each of the control charts and determine which, if any, of the out-of-control patterns are present.

Chapter 5: Analyzing Control Charts – What We Covered

Learning Objectives

You should now be able to describe:

- At least four of the common out-of-control detection rules
- How to analyze a control chart for out-of-control situations

Chapter Agenda

- Common Out-of-Control Detection Rules
- Patterns that Signal Out-of-Control
- **Breakout Exercise 5**

Chapter 6

Capability Analysis

Chapter 6: Capability Analysis – What We Will Cover

Learning Objectives

At the end of this chapter, you will be able to describe:

- The meaning of C_p, C_{pk}, P_p and P_{pk}
- The type of variation considered in the indices
- How to apply the formulas to calculate the indices

Chapter Agenda

- Capability Basics
- Bilateral Tolerances and Capability
- Unilateral Tolerances and Capability
- **Breakout Exercise 6**

QUALITY

CAPABILITY BASICS

Capability Basics

- Output of a stable process can be described by its statistical distribution.
- **The process must be stable (in statistical control) in order for the distribution to be useful for predicting future results.**
- Process centering and spread interact with respect to producing an acceptable product.
- A shift in process location, an increase in process spread or a combination of these factors may produce parts outside the specification limits.

Capability Basics

- Indices of process variation-only, relative to specifications: C_p and P_p
- Indices of **process variation** and **centering** combined, relative to specifications: C_{pk} and P_{pk}
- Ratios of process variation-only, relative to specifications: *CR* and *PR*

QUALITY

OMNEX

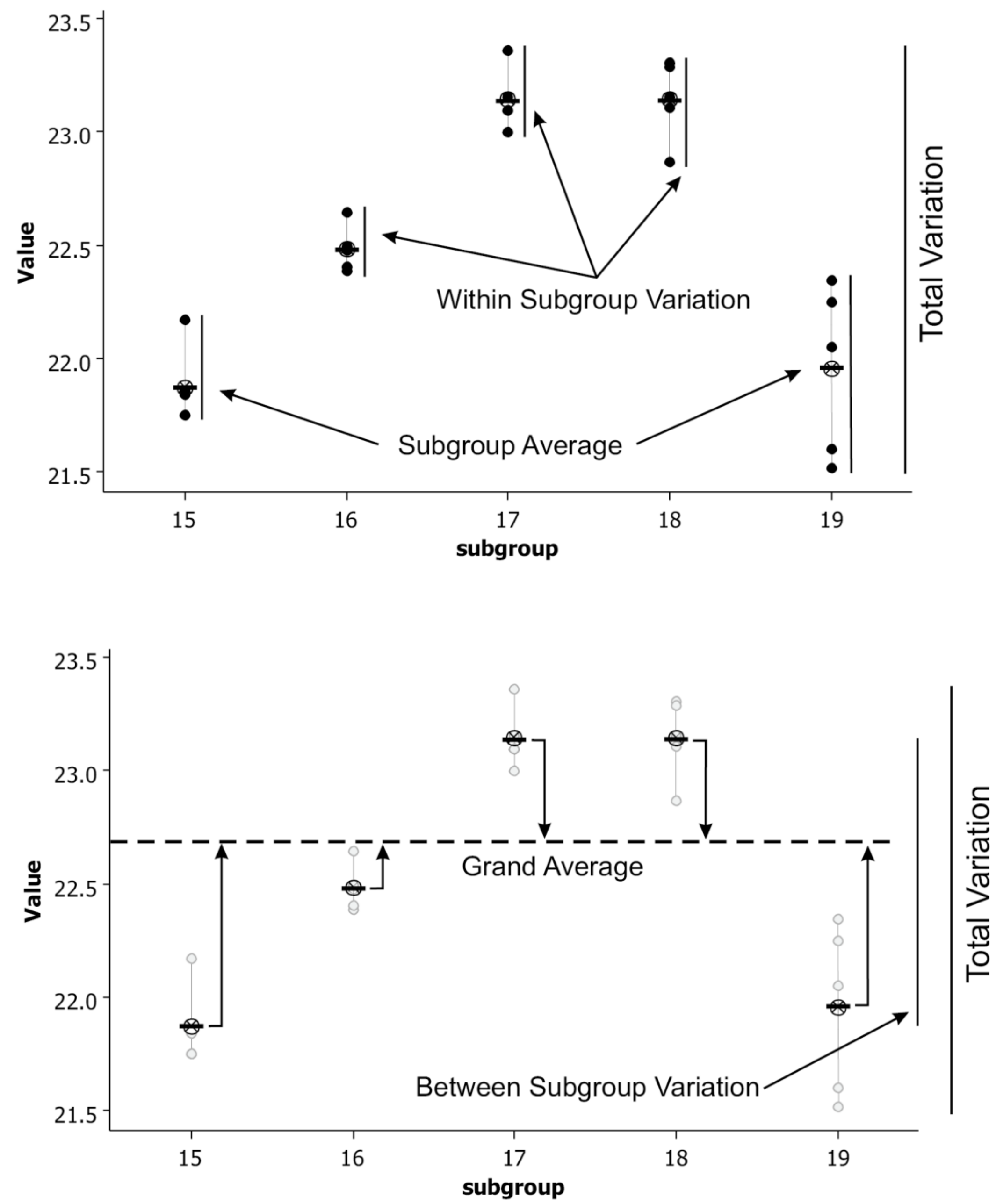
Capability Basics

Process variation and **process centering** are two separate process characteristics. Each needs to be understood separately from the other.

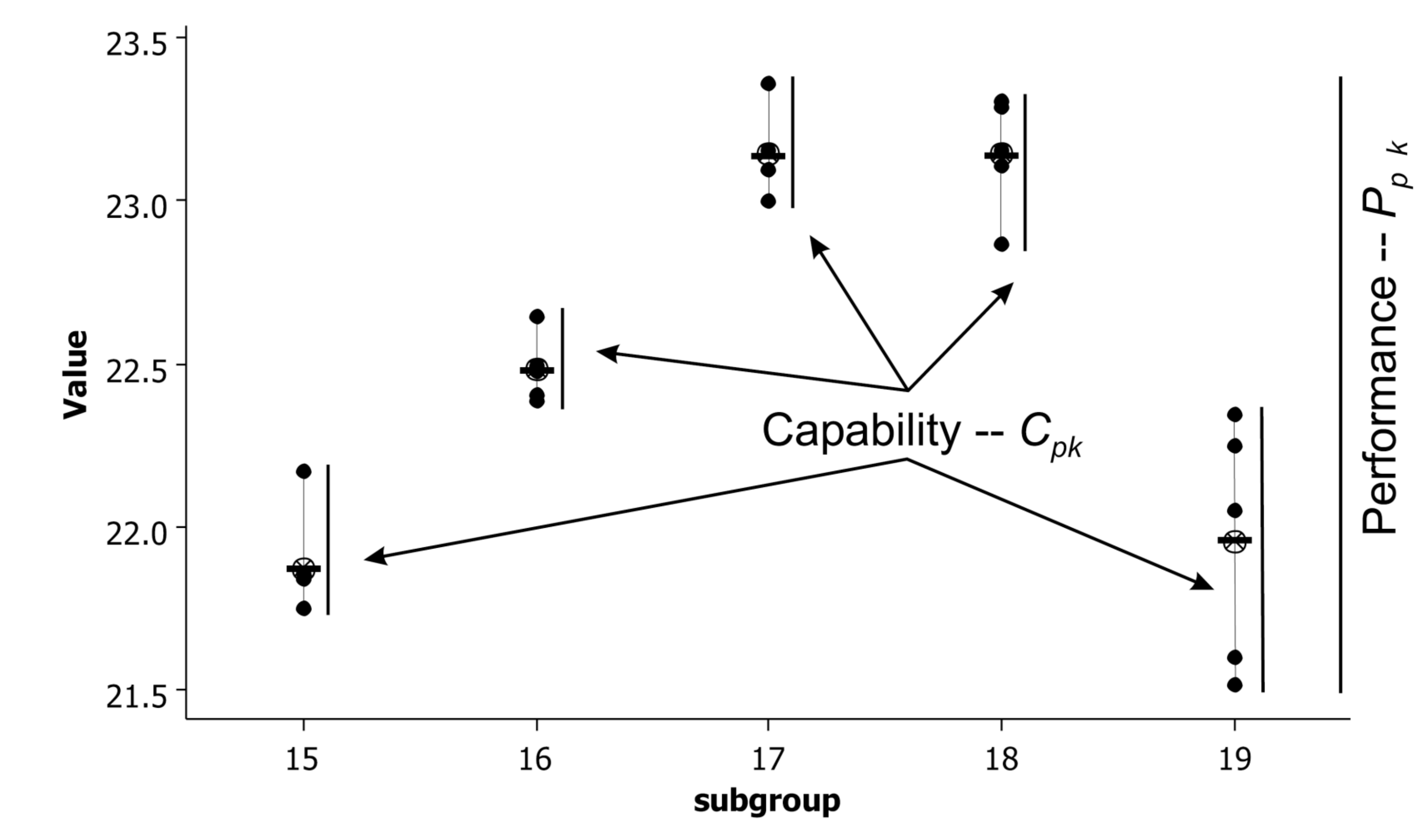
- To assist in analysis, it is convenient to combine the two characteristics into indices such as; C_p , C_{pk} or P_p , P_{pk}
- These indices can be useful for:
  - Measuring continual improvement using trends over time.
  - Prioritizing the order in which processes will be improved.
  - **These indices should always be evaluated in pairs.**

$$C_p , C_{pk} \quad \text{or} \quad P_p , P_{pk}$$

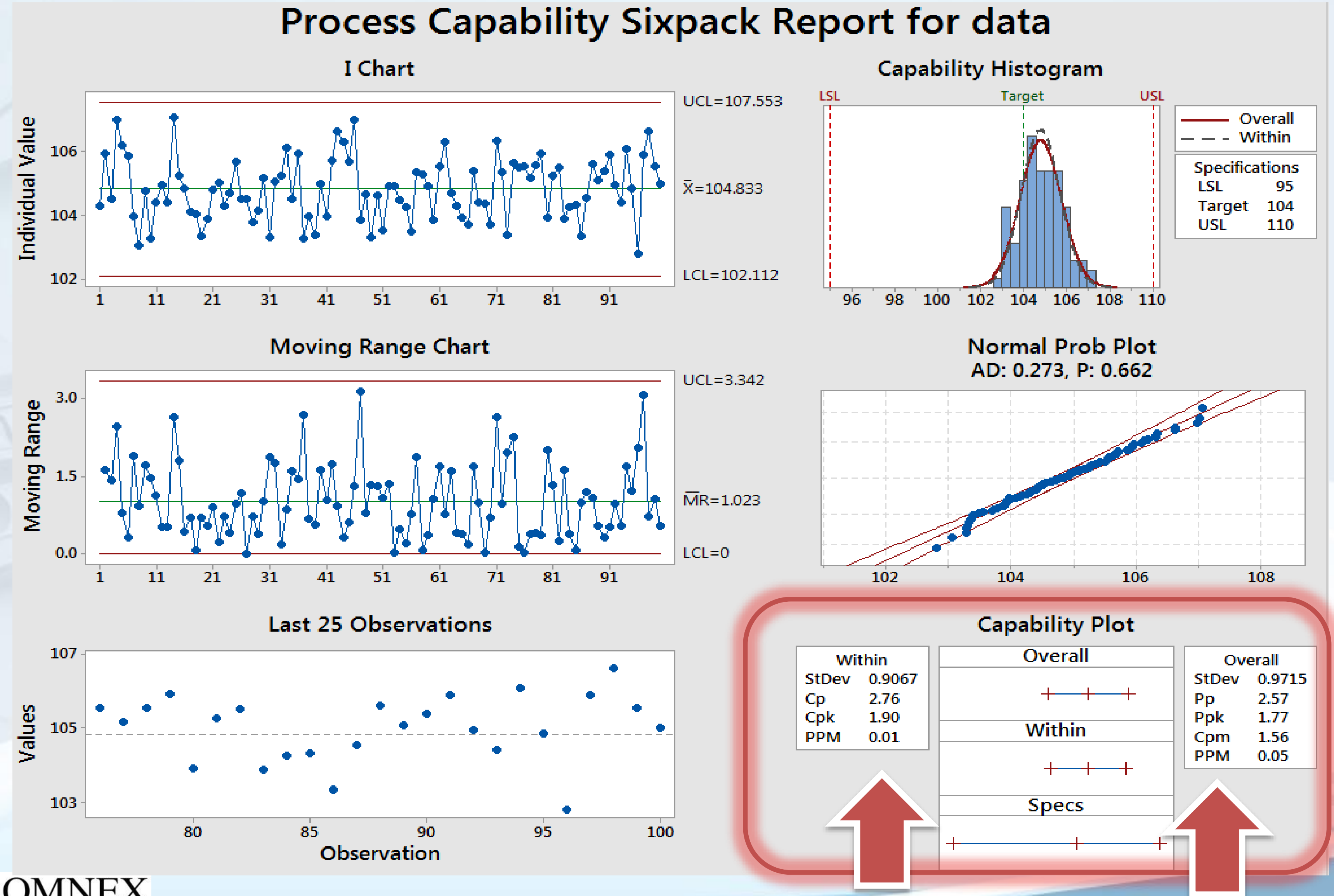
Capability Basics
Within & Between



Capability Basics



Best Practice: Evaluate the 4 Indices Together

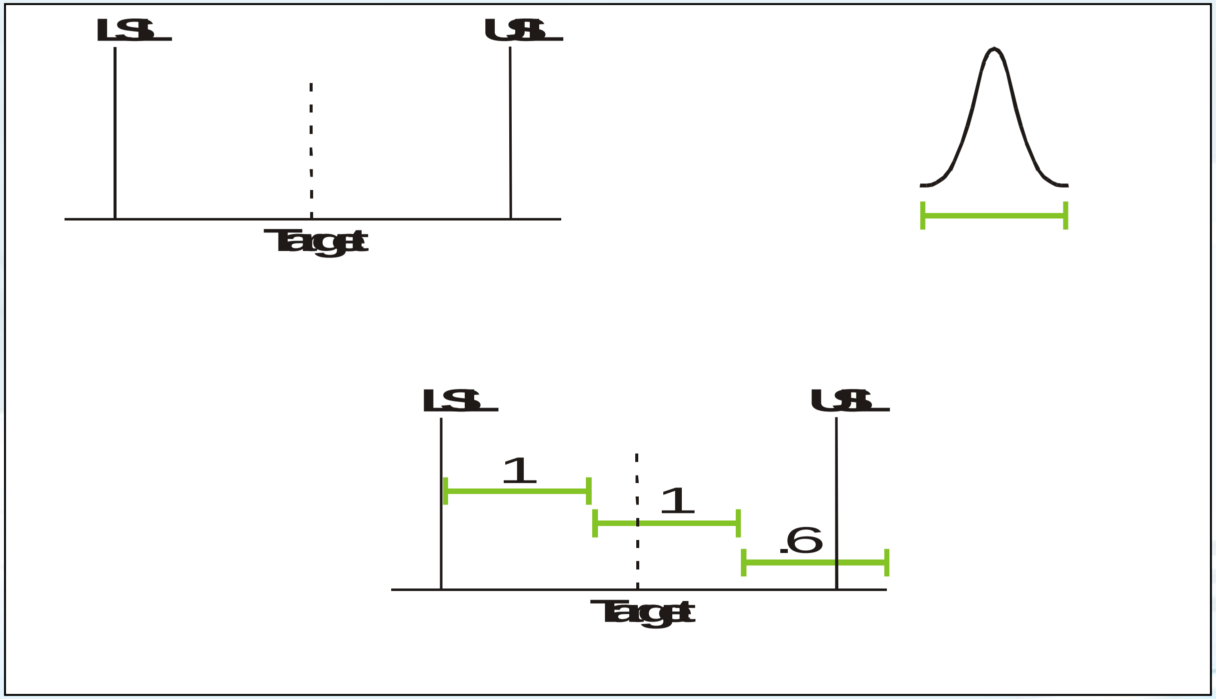


BILATERAL TOLERANCES

Process Capability and Performance Indices

Bilateral Tolerances

- C_p is the number of times the process distribution (Six Sigma range) can "go into" the tolerance range.



Bilateral Tolerances

- C_p : This is a capability index. It compares the process capability to the maximum allowable variation as indicated by the tolerance.
  - Provides a measure of how well the process will satisfy the variability requirements.
  - C_p is calculated by

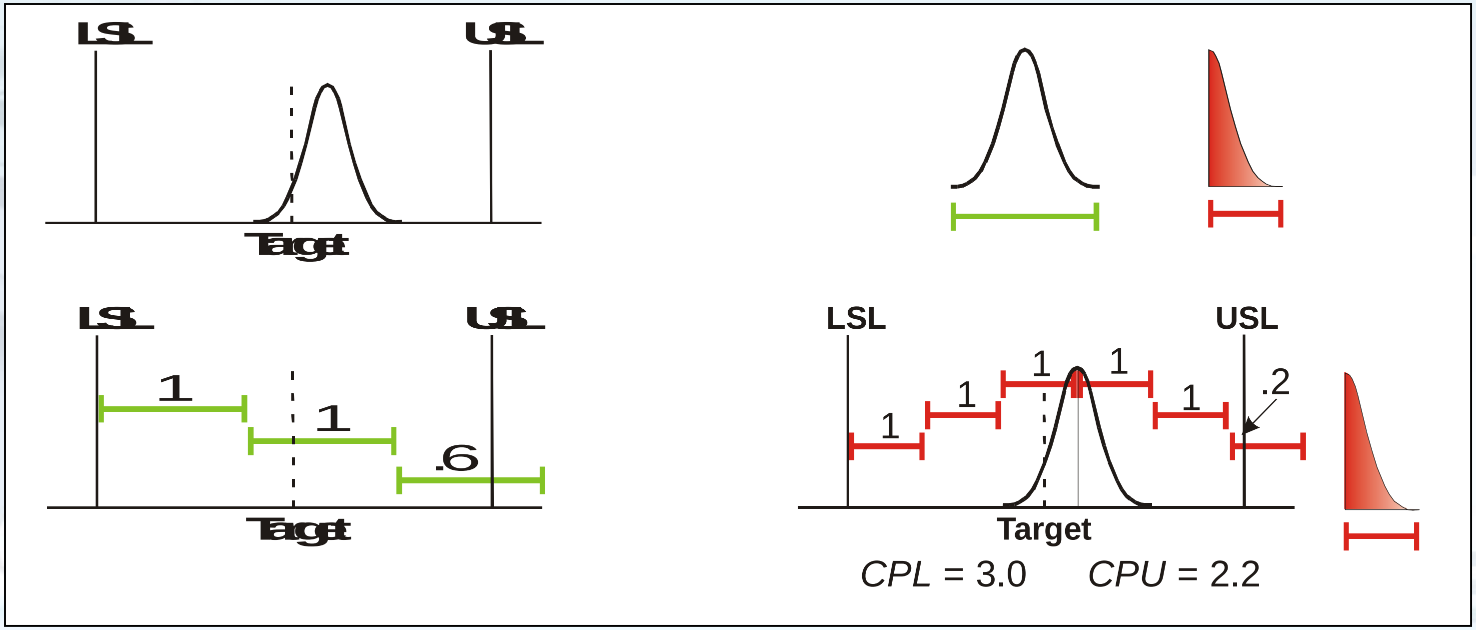
$$C_p = \frac{USL - LSL}{6\sigma_C} = \frac{USL - LSL}{6\left(\bar{R}/d_2\right)}$$

- C_p is not impacted by the process location – this index can be calculated only for two-sided (bilateral) tolerances.

OMNEX

Bilateral Tolerances

- C_{pk} is the number of times half of the process distribution (Three Sigma range) can "go into" the tolerance range, starting at the process average.



Bilateral Tolerances

- C_{pk} : This is a capability index. It takes the process location as well as the capability into account.

- For bilateral tolerances, C_{pk} will always be less than or equal to C_p
  - C_{pk} will be equal to C_p only if the process is centered
  - C_{pk} is calculated as the **minimum** of *CPU* or *CPL*

$$CPU = \frac{USL - \bar{\bar{X}}}{3\sigma_C} = \frac{USL - \bar{\bar{X}}}{3\left(\bar{R}/d_2\right)} \qquad CPL = \frac{\bar{\bar{X}} - LSL}{3\sigma_C} = \frac{\bar{\bar{X}} - LSL}{3\left(\bar{R}/d_2\right)}$$

Bilateral Tolerances

- P_p : This is a performance index. It compares the process performance to the maximum allowable variation as indicated by the tolerance.
  - This index provides a measure of how well the process will satisfy the variability requirements.
  - P_p is calculated by

$$P_p = \frac{USL - LSL}{6\sigma_P} = \frac{USL - LSL}{6s}$$

- P_p is not impacted by the process location – this index can be calculated only for two-sided (bilateral) tolerances.

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Bilateral Tolerances

- P_{pk} : This is a performance index. It takes the process location as well as the performance into account.

- For bilateral tolerances P_{pk} will always be less than or equal to P_p
  - P_{pk} will be equal to P_p only if the process is centered
  - P_{pk} is calculated as the **minimum** of *PPU* or *PPL*

$$PPU = \frac{USL - \bar{\bar{X}}}{3\sigma_P} = \frac{USL - \bar{\bar{X}}}{3s} \qquad PPL = \frac{\bar{\bar{X}} - LSL}{3\sigma_P} = \frac{\bar{\bar{X}} - LSL}{3s}$$

Bilateral Tolerances

- P_{pk} and P_p should always be evaluated and analyzed together.
- A P_p value significantly greater than the corresponding P_{pk} indicates an opportunity for improvement by centering the process.

Bilateral Tolerances

- ***CR***: This is the capability ratio and is simply the reciprocal of C_p

  CR = $1/C_p$

- ***PR***: This is the performance ratio and is simply the reciprocal of P_p

  PR = $1/P_p$

Bilateral Tolerances

- A parts-per-million (ppm) nonconformance rate is sometimes used as a supplemental measure of process capability.
- To estimate the nonconformance rate using capability index information, a probability distribution of the data must be defined.
  - While the normal distribution often is used for this purpose, this is an assumption that should be validated before proceeding further.
  - The nonlinear relationship between the capability index and the proportion nonconforming should be understood in order to make correct inferences.

UNILATERAL TOLERANCES

Unilateral Tolerances

- C_p : This is a capability index. It compares the process capability to the maximum allowable variation as indicated by the tolerance.
  - ***This index has no meaning for unilateral tolerances.***

- P_p : This is a performance index. It compares the process performance to the maximum allowable variation as indicated by the tolerance.
  - ***This index has no meaning for unilateral tolerances.***

- If the product characteristic has a physical limit (e.g., flatness cannot be less than zero), a C_p (or P_p) could be calculated using the physical limit (0.0) as a surrogate lower limit – but, this number will not have the same relationship to C_{pk} (P_{pk}) as it does in the bilateral case.

Unilateral Tolerances

- C_{pk} is directly related to the proportion nonconforming produced by the process – it is equal to *CPU* or *CPL* depending whether the tolerance is a *USL* or a *LSL* where:

$$CPU = \frac{USL - \bar{\bar{X}}}{3\left(\bar{R}/d_2\right)} \qquad CPL = \frac{\bar{\bar{X}} - LSL}{3\left(\bar{R}/d_2\right)}$$

- With unilateral tolerances with a physical limit, C_{pk} can be less than, equal to, or greater than C_p

Unilateral Tolerances

- P_{pk} is directly related to the proportion nonconforming produced by the process – it is equal to *PPU* or *PPL* depending whether the tolerance is a *USL* or a *LSL* where:

$$PPU = \frac{USL - \bar{\bar{X}}}{3s} \qquad PPL = \frac{\bar{\bar{X}} - LSL}{3s}$$

- With unilateral tolerances with a physical limit, P_{pk} can be less than, equal to or greater than P_p

Unilateral Tolerances

- *CR*: This is the capability ratio and is simply the reciprocal of C_p
  - ***As such, this index has no meaning for unilateral tolerances.***

- *PR*: This is the performance ratio and is simply the reciprocal of P_p
  - ***As such, this index has no meaning for unilateral tolerances.***

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Breakout Exercise 6

Calculating Indices

Breakout Exercise 6: Calculating Indices

Instructions

- Turn to Exercise 6 in your workbook.
- Calculate C_p , C_{pk} and P_p , P_{pk} using the Brinell Data found in Exercise 1 and the control charts completed in Exercise 3.

As time allows, the instructor will provide additional problems to solve

Chapter 6: Capability Analysis – What We Covered

Learning Objectives

You should now be able to describe:

- The meaning of C_p , C_{pk} , P_p and P_{pk}
- The type of variation considered in the indices
- How to apply the formulas to calculate the indices

Chapter Agenda

- Capability Basics
- Bilateral Tolerances and Capability
- Unilateral Tolerances and Capability
- **Breakout Exercise 6**

Thank You!

Questions?

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